Scientific Computing

March 3, 2025

Announcements

→ HW 3 due Wednesday at 11:59pm

→ Wednesday is also the in-person midterm exam

→ Friday, no lecture, extra office hours while you work on take-home (time TBD)

Any review questions.

Today

→ Branch and Bound

→ Object-Oriented Programming

Office Hours:

Mon + Fri

9:30am - 10:30am

Cudahy 307

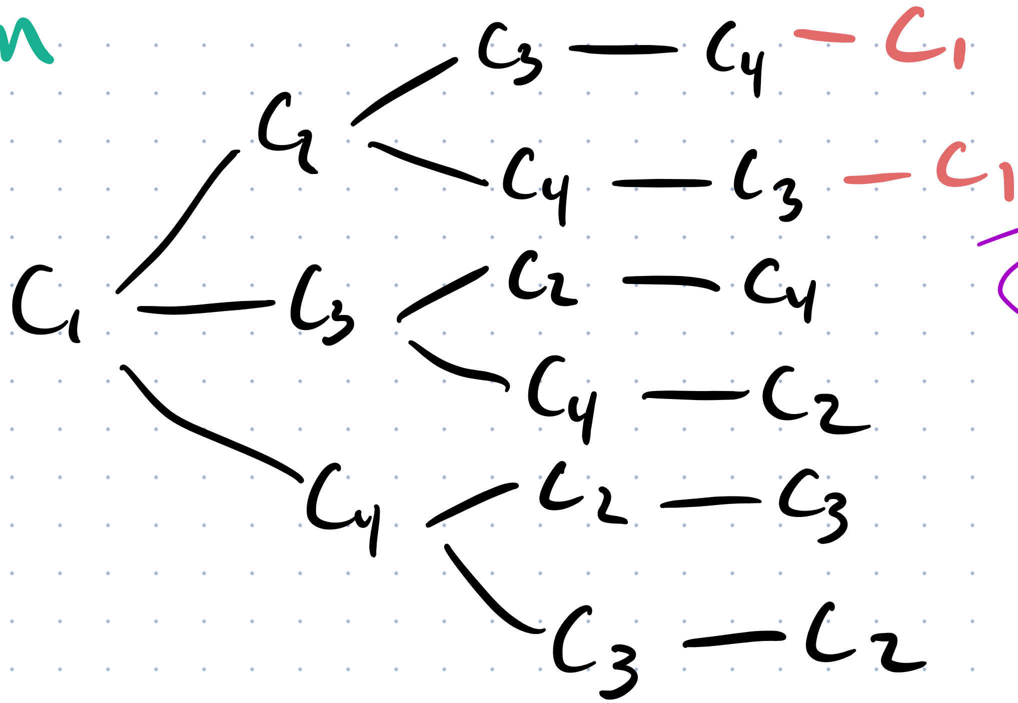
One more example – traveling salesman

* Pick a start city C_1. (assume 4 cities)

Branch: Next city to visit that hasn't been visited yet.

cities $\{C_1, C_2, C_3, C_4\}$

Minimization problem



For pruning:

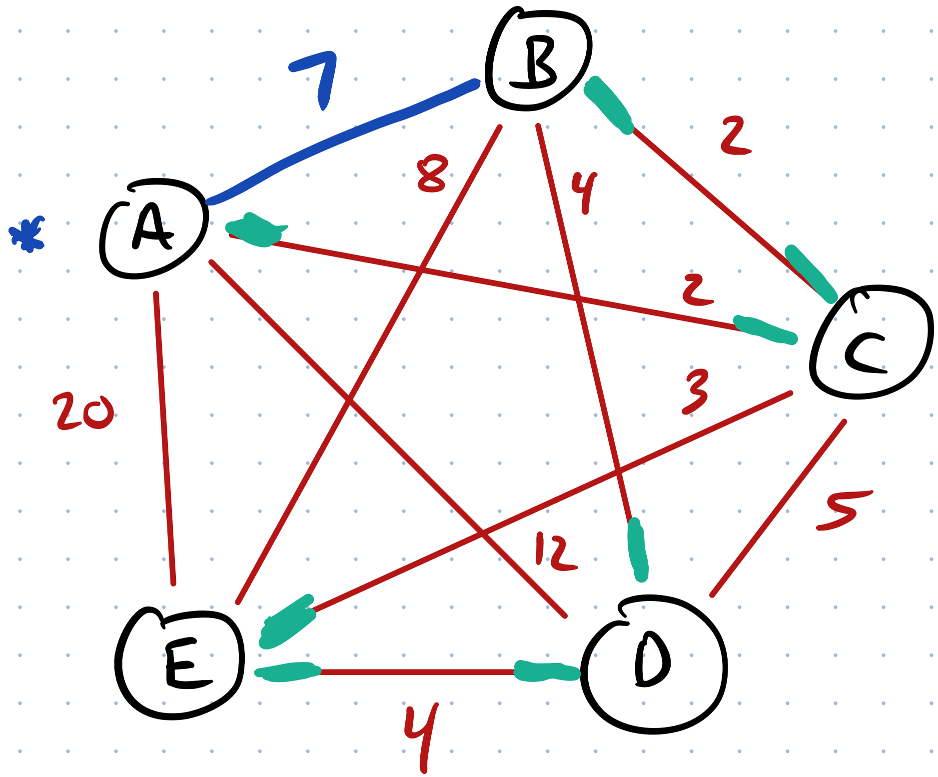
Lower bound

or

upper bound

?

* Lower bound: On the branch A→B

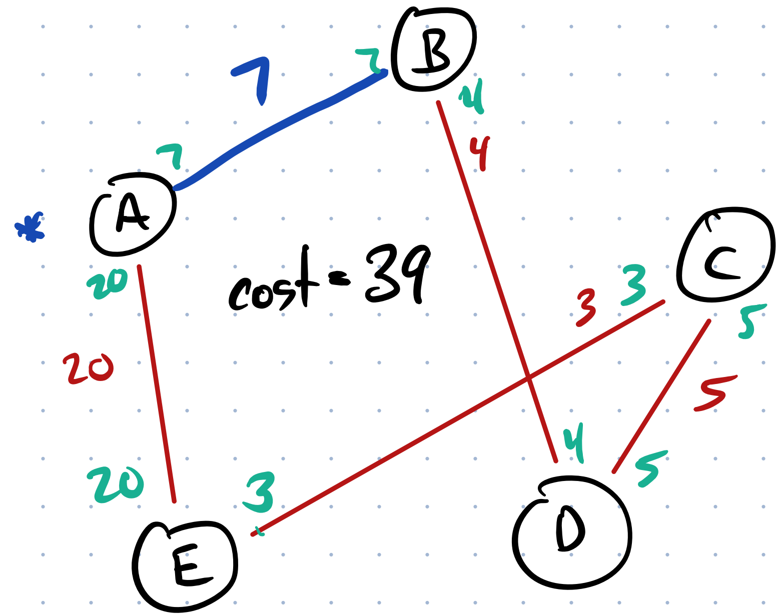


We're going to have to exit B. Cheapest way: 2.
We're going to have to enter and exit C. Cheapest: 4
D: 8
E: 7
Back into A: 2

Wrong: lower bound = 7 + 2 + 4 + 8 + 7 + 2 = 30
Double counts! When you exit B, you enter some other node. When you exit C, you enter some other node, etc.

Let T be a given tour (a solution).

If you add up the cost going into and out of each city, you get <u>double</u> the cost, because you're counting each edge twice.



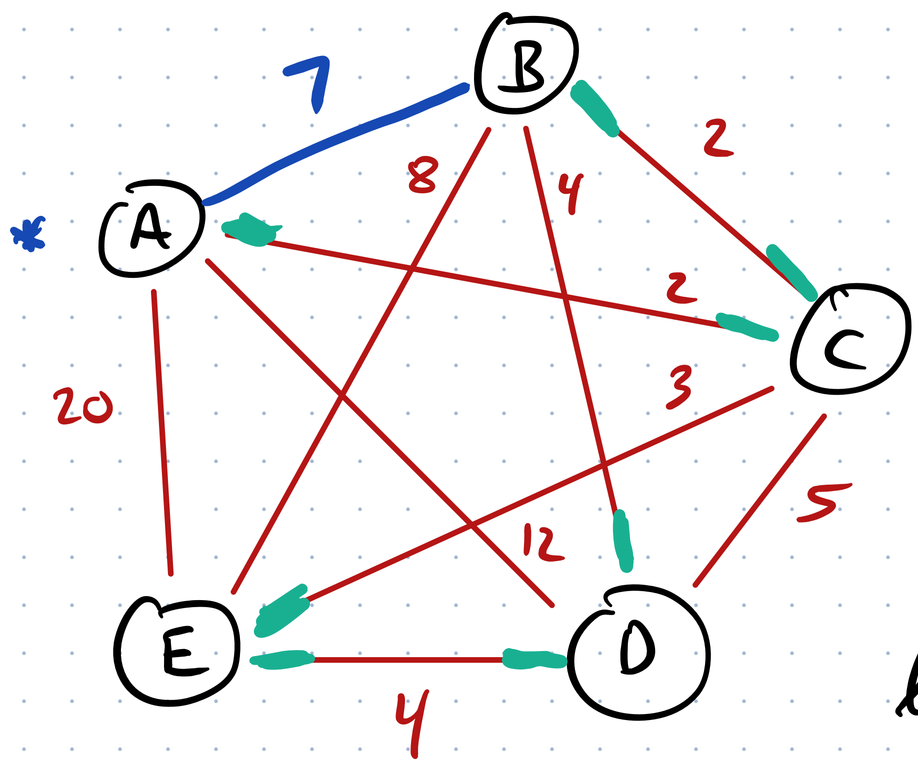
$$\mathrm{cost}(T) = \frac{1}{2} \cdot \sum_{v \in V} \Big([\text{cost to enter } v] + [\text{cost to exit } v] \Big)$$

sum over each vertex v

Now suppose we're in some subspace S and we want a lower bound on the cost of any tour in S. Let $T \in S$ be arbitrary.

$$\mathrm{cost}(T) = \frac{1}{2}\Big(\underbrace{[\text{enter } v_1] + [\text{exit } v_1]}_{\substack{\geq \text{ sum of two} \\ \text{cheapest edges} \\ \text{attached to } v_1}} + \cdots + \underbrace{[\text{enter } v_n] + [\text{exit } v_n]}_{\substack{\geq \text{ sum of two} \\ \text{cheapest edges} \\ \text{attached to } v_n}}\Big)$$

$$\geq \frac{1}{2}\Big(\text{sum of: for each vertex, use any edges already decided on, plus cheapest remaining, to get two total}\Big)$$



chosen

$$\frac{1}{2}\Big(\underbrace{\overset{*}{7}+2}_{A} + \underbrace{\overset{*}{7}+2}_{B} + \underbrace{2+2}_{C} + \underbrace{4+4}_{D} + \underbrace{4+3}_{E}\Big)$$

$$= \frac{1}{2}(37) = 18.5$$

lower bound: 19

Branch where $A \to B$

Every sol that starts $A \to B$ has a final cost of ≥ 18.5

≥ 19

B+B tree time!

Greedy: $A \to C \to B \to D \to E \to A$

$= 2+2+4+4+20 = 32$

Another: $B \to C \to A \to D \to E \to B$

$= 2+2+12+4+8 = 28$

$C \to A \to B \to D \to E \to C$

$= 2+7+4+4+3 = 20$!