MATH 2350 – HOMEWORK 6

Fall 2025

due Wednesday, **December 3**, at the beginning of class

Sections 3.3, 4.1, 4.2

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. It may be handwritten (as long as I can read it) or typed with software such as Word or Latex. Please write the questions in the correct order. Explain all reasoning.

You are no longer required to show your scratch work. However, doing scratch work on another paper before trying to write your proof is still the way to succeed on these problems!

1. Prove the following set inequality:
$$(\{n^2 - 1 : n \in \mathbb{Z}\} \cap \{2k : k \in \mathbb{N}\}) \subseteq \{4m : m \in \mathbb{Z}\}.$$
2. Prove the following set inequality:
$$(\{6k + 1 : k \in \mathbb{Z}\} \cup \{6m - 1 : m \in \mathbb{Z}\}) \subseteq \{2n + 1 : n \in \mathbb{Z}\}.$$
3. Prove that $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.
4. Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.
5. Draw the two-sided arrow diagram for the function $f : \mathcal{P}(\{4, 5, 6\}) \to \mathcal{P}(\{1, 2, 3\})$ defined by
$$f(S) = \{x - 3 : x \in S\} \smallsetminus \{2\}.$$
6. Prove that the function $h : \mathbb{N} \to \mathbb{N}$ defined by $h(n) =$ [the sum of the digits in n (in base 10)] is surjective. Prove that it's not injective.
7. Let $c : \mathcal{P}(\{x, y, z\}) \to \mathcal{P}(\{x, y, z\})$ be the function with the rule $c(A) = \{x, y, z\} \smallsetminus A$, and let $n : \mathcal{P}(\{x, y, z\}) \to \{0, 1, 2, 3\}$ be the function such that $n(A)$ is the number of elements in the set A. Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the two-sided arrow diagram.
8. (a) Suppose $f : Q \to R$ and $g : R \to S$ are both injective (one-to-one) functions. Prove that $g \circ f : Q \to S$ is injective.
   (b) Suppose $f : Q \to R$ and $g : R \to S$ are both surjective (onto) functions. Prove that $g \circ f : Q \to S$ is surjective.
   (c) Suppose $f : Q \to R$ and $g : R \to S$ are both bijective functions. Prove that $g \circ f : Q \to S$ is bijective.
9. Let $A = \{0, 1, 2, 3\}$ and let $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$ be the set of binary strings with three digits. Define $g : B \to A$ by $g(s) =$ [the number of 1s in s]. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).