MATH 2350 – HOMEWORK 3

Fall 2025

due Wednesday, **October 15**, at the beginning of class

Sections 2.1, 2.2

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. It may be handwritten (as long as I can read it) or typed with software such as Word or Latex. Please write the questions in the correct order. Explain all reasoning.

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1. Prove that if a and b are nonzero rational numbers, then so is $\dfrac{ab}{2} + \dfrac{1}{b}$.
2. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If x, y, and z are integers and if x divides y and x divides z, then x^2 divides yz.
3. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If x, y, and z are integers and if x divides z and y divides z, then xy divides z.
4. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $3^n + 1$ is divisible by 5.
5. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $n^3 + 2n$ is divisible by 4.
6. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If m is a positive odd integer, then $m^2 - 1$ is divisible by 8.
7. Prove that the sum of any three consecutive integers (for example, $6 + 7 + 8$) is always a multiple of 3.
8. Prove that if 3 divides $4^{n-1} - 1$ then 3 divides $4^n - 1$.
9. Prove that no perfect square can have the form $3n + 2$ for an integer n.
10. Prove that if n is an even integer, then $4(n + 1) + 3$ is odd.