

Template for proofs by induction of summation formulas

Theorem:

Proof:

Define $P(n) =$ “”.

The base case is $P(1)$ which says “”,
which is true because

For the induction step, let $n \geq 1$. We will assume $P(n)$, which says

$P(n) =$ “”.

and we will prove $P(n+1)$, which says

$P(n+1) =$ “”.

The LHS of $P(n+1)$ is

$$\begin{aligned}\sum_{k=1}^{n+1} k &= \left(\sum_{k=1}^n k \right) + (n+1) \\ &= \left(\frac{n(n+1)}{2} \right) + (n+1),\end{aligned}$$

where the second equality is by the induction hypothesis. Now, by arithmetic

which is the RHS of $P(n+1)$. Therefore, by the principle of mathematical induction, we have proved the theorem.
 $\square$