Math 1450 - Calculus 1

Wed, Dec 3

Announcements:

* HW 13 due tomorrow
* Help Desk hours end on Friday
* Final Exam:
  Wednesday, Dec 10, 8pm - 10pm
  Weaster Auditorium

Friday lecture: Review
Bring Qs!

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 5.4: Theorems about definite integrals
Review!

Section 5.4: Theorems about Definite Integrals

Fact #1: If f is continuous and a and b are any #s:

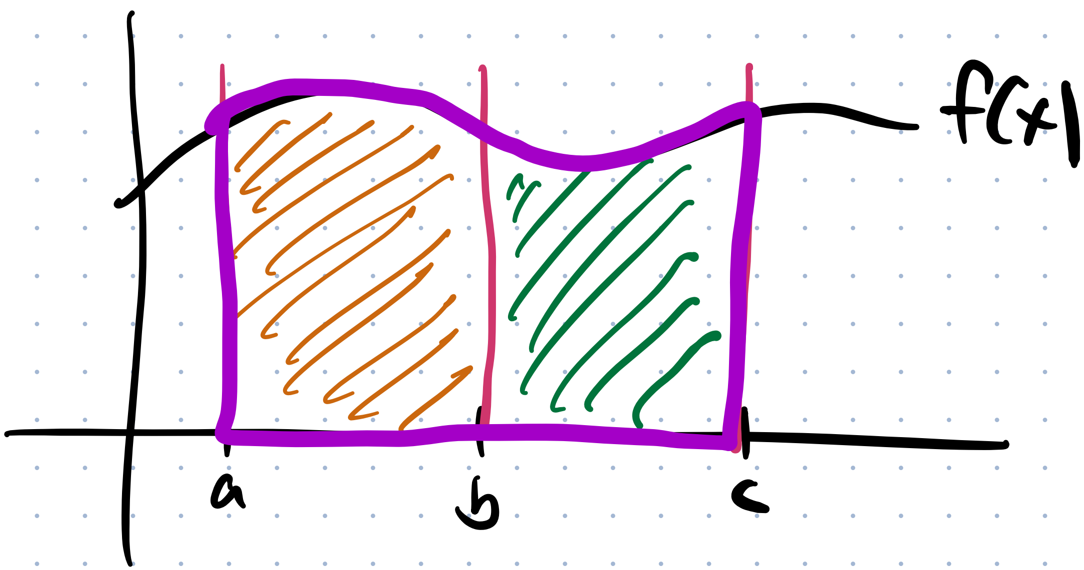
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Ex: If $\int_3^5 f(x)\,dx = 10$, then $\int_5^3 f(x)\,dx = -10$.

Fact #2: If f is continuous and a, b, c are any #s, then:

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

orange area + green area = purple area



Fact #3

Let f and g be continuous functions.

Then,

$$\int_a^b (f(x)+g(x))\,dx = \left[\int_a^b f(x)\,dx\right] + \left[\int_a^b g(x)\,dx\right]$$

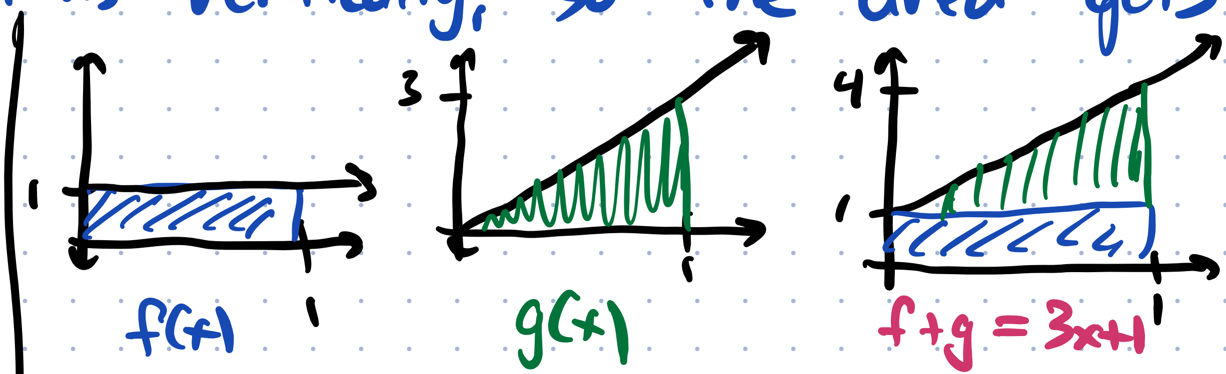
$$\left(\text{Like } \frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))\right)$$

Geometrically: $f(x)+g(x)$ is like stacking the two functions vertically, so the area gets added

$f(x)=1$

$g(x)=3x$

$a=0,\ b=1$



Fact #4

Let f be continuous and let c be any constant.

Then,

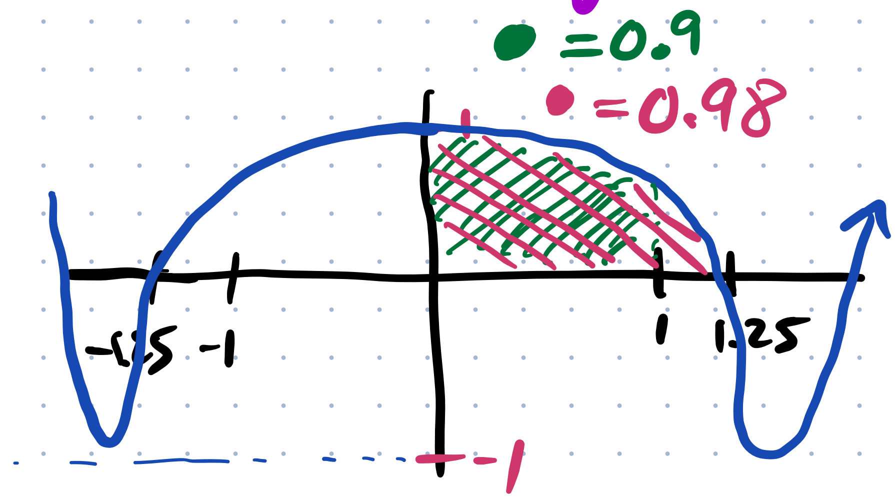
$$\int_a^b (c \cdot f(x))\,dx = c \cdot \int_a^b f(x)\,dx$$

Geometrically, the area under a vertically stretched function grows by the same multiplier.

If $\int_2^7 g(x)\,dx = 3$, what is $\int_2^7 -2 \cdot g(x)\,dx$?

-6

Ex: Given that $\int_0^{1.25} \cos(x^2)\,dx \approx 0.98$ and $\int_0^{1} \cos(x^2)\,dx \approx 0.9$, calculate the following:



(a) $\int_1^{1.25} \cos(x^2)\,dx$

Fact # 4:

$$\int_0^{1} \cos(x^2)\,dx + \int_1^{1.25} \cos(x^2)\,dx = \int_0^{1.25} \cos(x^2)\,dx$$

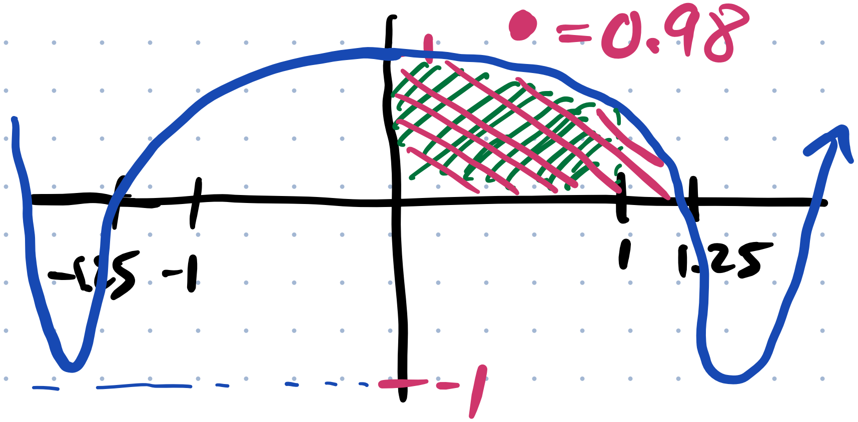
$$0.9 + ? = 0.98$$

$$\boxed{0.08}$$

Ex: Given that $\int_0^{1.25} \cos(x^2)\,dx \approx 0.98$ and

$\int_0^{1} \cos(x^2)\,dx \approx 0.9$, calculate the following:

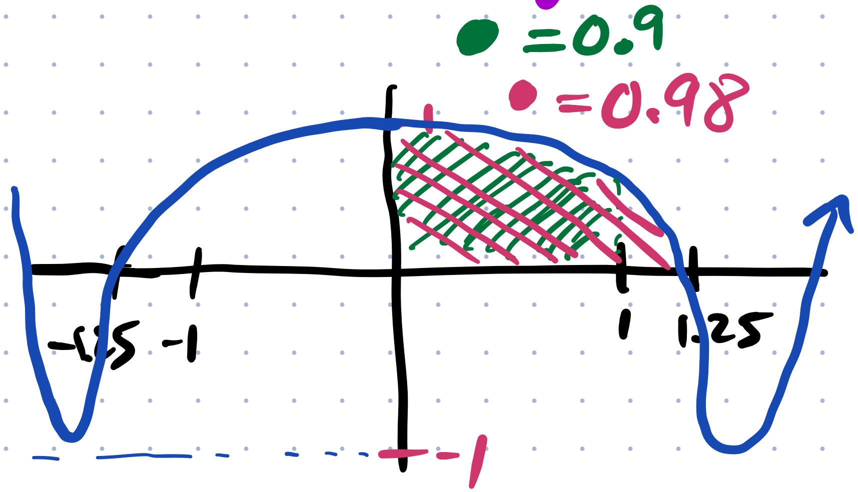
Assume you know the graph is symmetric over the y-axis.



$$\int_{-1}^{1} \cos(x^2)\,dx = \underbrace{\int_{-1}^{0} \cos(x^2)\,dx}_{0.9 \text{ by symmetry}} + \underbrace{\int_{0}^{1} \cos(x^2)\,dx}_{0.9,\ \text{given to us}}$$

$= 1.8$

Ex: Given that $\int_0^{1.25} \cos(x^2)\,dx \approx 0.98$ and $\int_0^{1} \cos(x^2)\,dx \approx 0.9$, calculate the following:



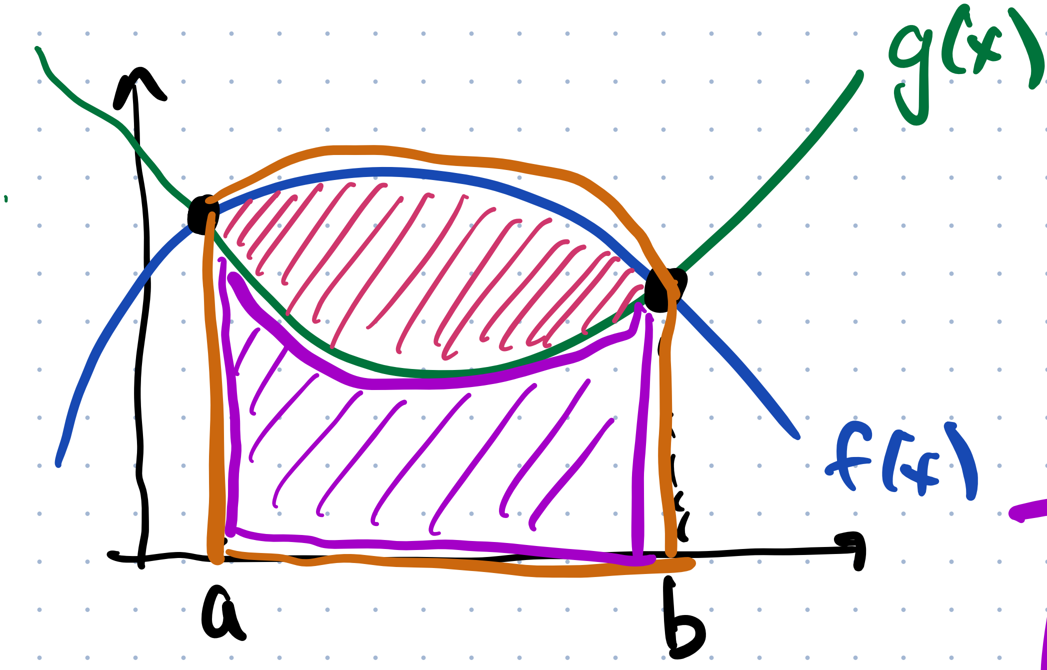
Assume you know the graph is symmetric over the y-axis.

$$\int_{1.25}^{-1} \cos(x^2)\,dx = -\int_{-1}^{1.25} \cos(x^2)\,dx = -\left(\int_{-1}^{0} \cos(x^2)\,dx + \int_{0}^{1.25} \cos(x^2)\,dx\right)$$

$$= -(0.9 + 0.98) = \boxed{-1.88}$$

Area between two functions

To calculate the area between two curves, we use the integral of their difference.

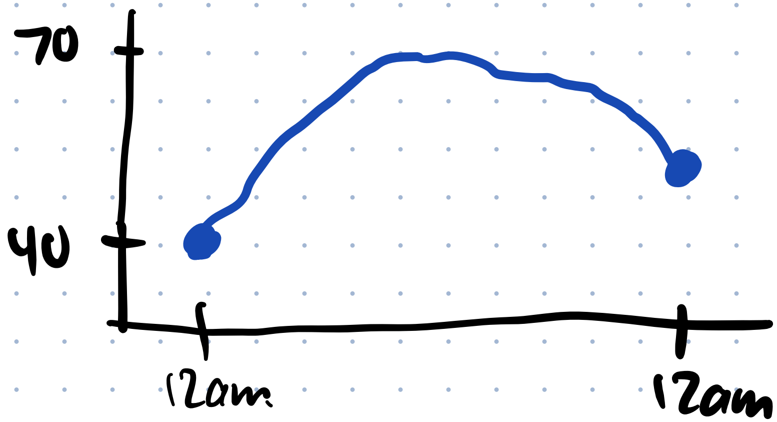


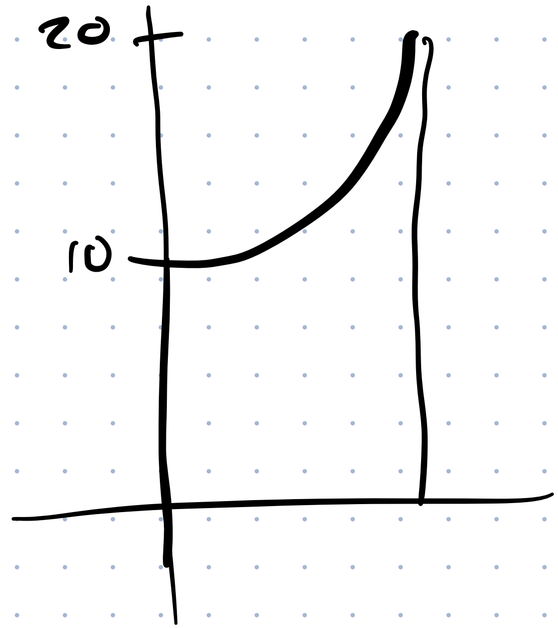
$$\underbrace{\int_a^b f(x)\,dx}\; - \;\underbrace{\int_a^b g(x)\,dx}$$

$$\boxed{= \int_a^b \big(f(x) - g(x)\big)\,dx}$$

Average Value of a function

If the temperature over a day looks like this:





how do we compute the average temperature?

Average value of $f(x)$ between $x=a$ and $x=b$ is:

$$Avg = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx$$