Math 1450 – Calculus 1

Fri, Nov. 21

Announcements:

* HW 12 was due last night – can do up to 3 days late for half credit
* Next Week: lecture Monday
  discussion Tuesday
  nothing Wed, Thurs, Fri
* Final Exam:
  Wednesday, Dec 10, 8pm – 10pm
  Weaster Auditorium

Office Hours

Mondays, 12–1

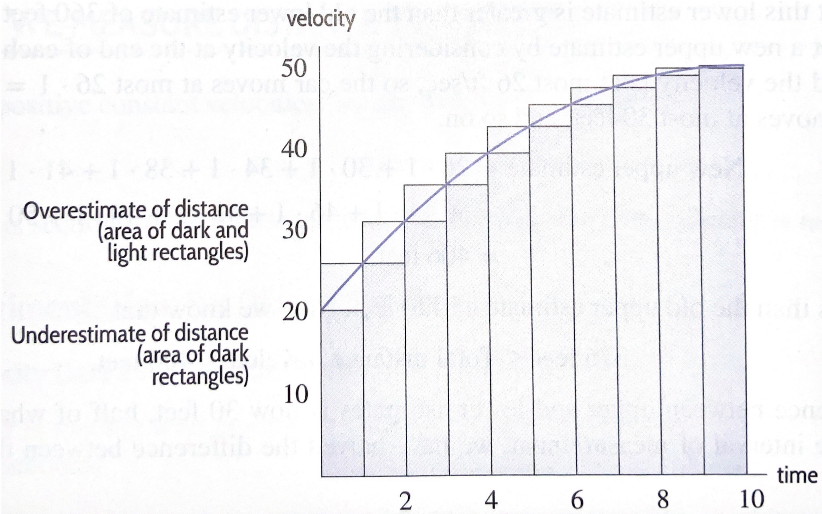
Wednesdays, 2–3

+ Help Desk! 12–1

Today:

→ 5.1: How do we measure distance traveled?

→ 5.2: The definite integral



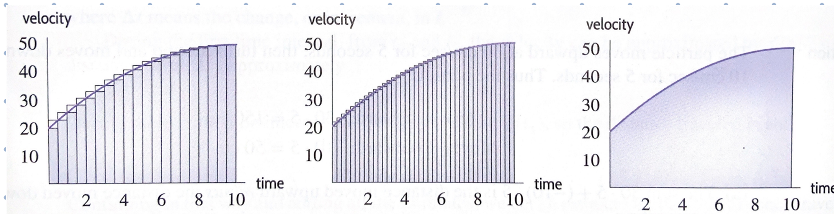


Figure 5.3: Velocity measured every 1/2 second

Figure 5.4: Velocity measured every 1/4 second

Figure 5.5: Distance traveled is area under curve

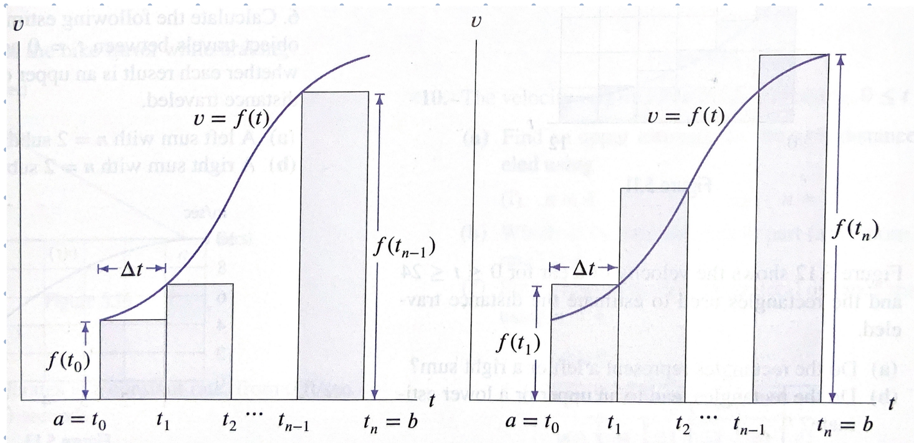
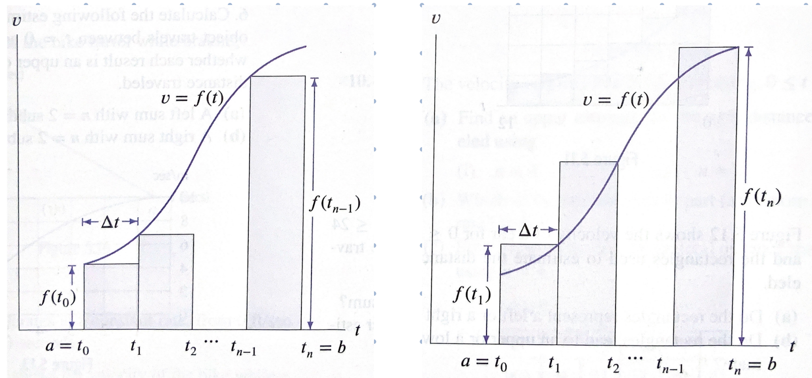


Figure 5.8: Left-hand sums

Figure 5.9: Right-hand sums



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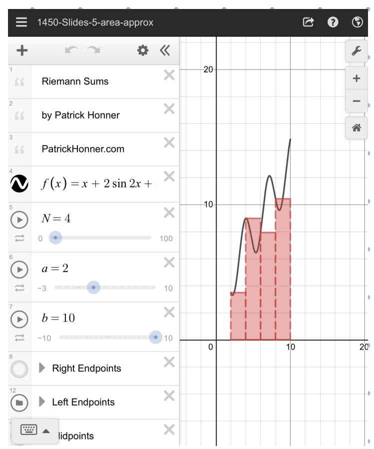
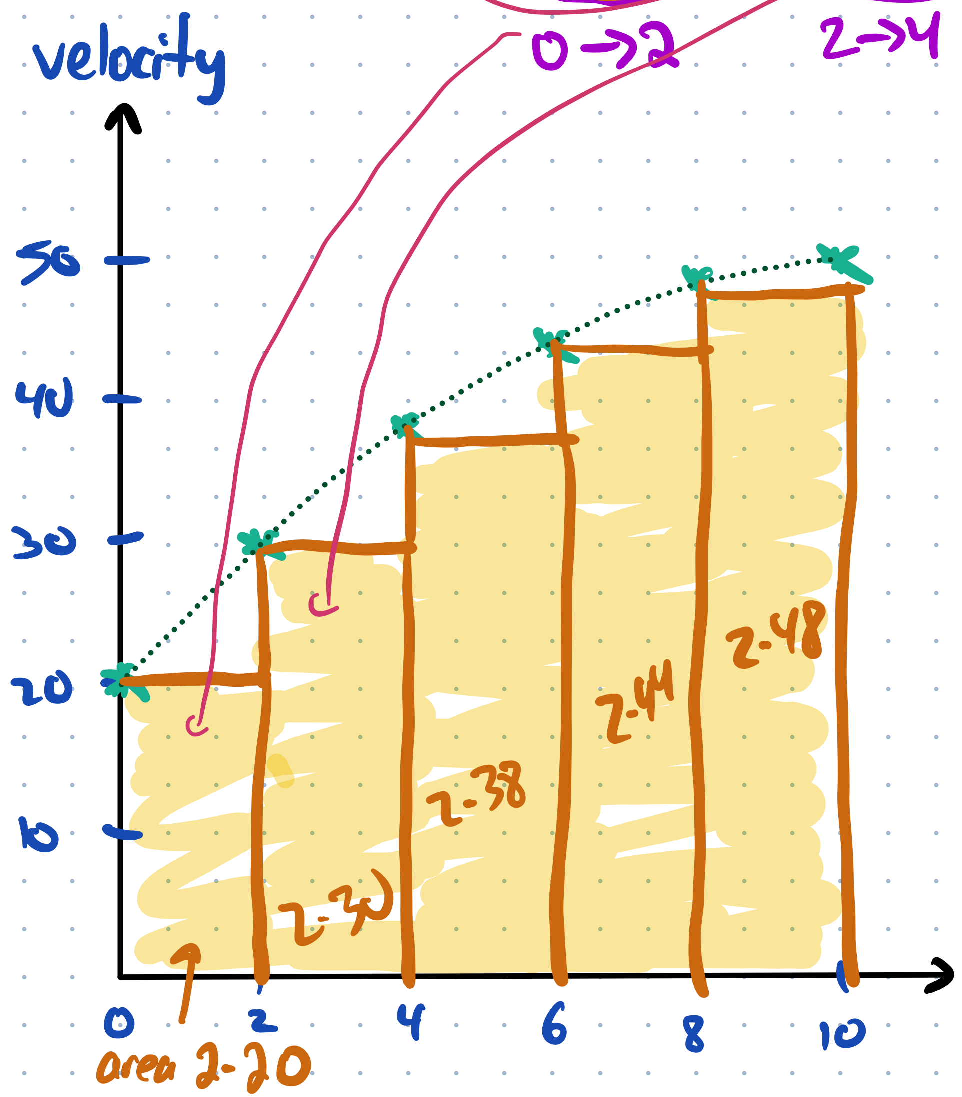


time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

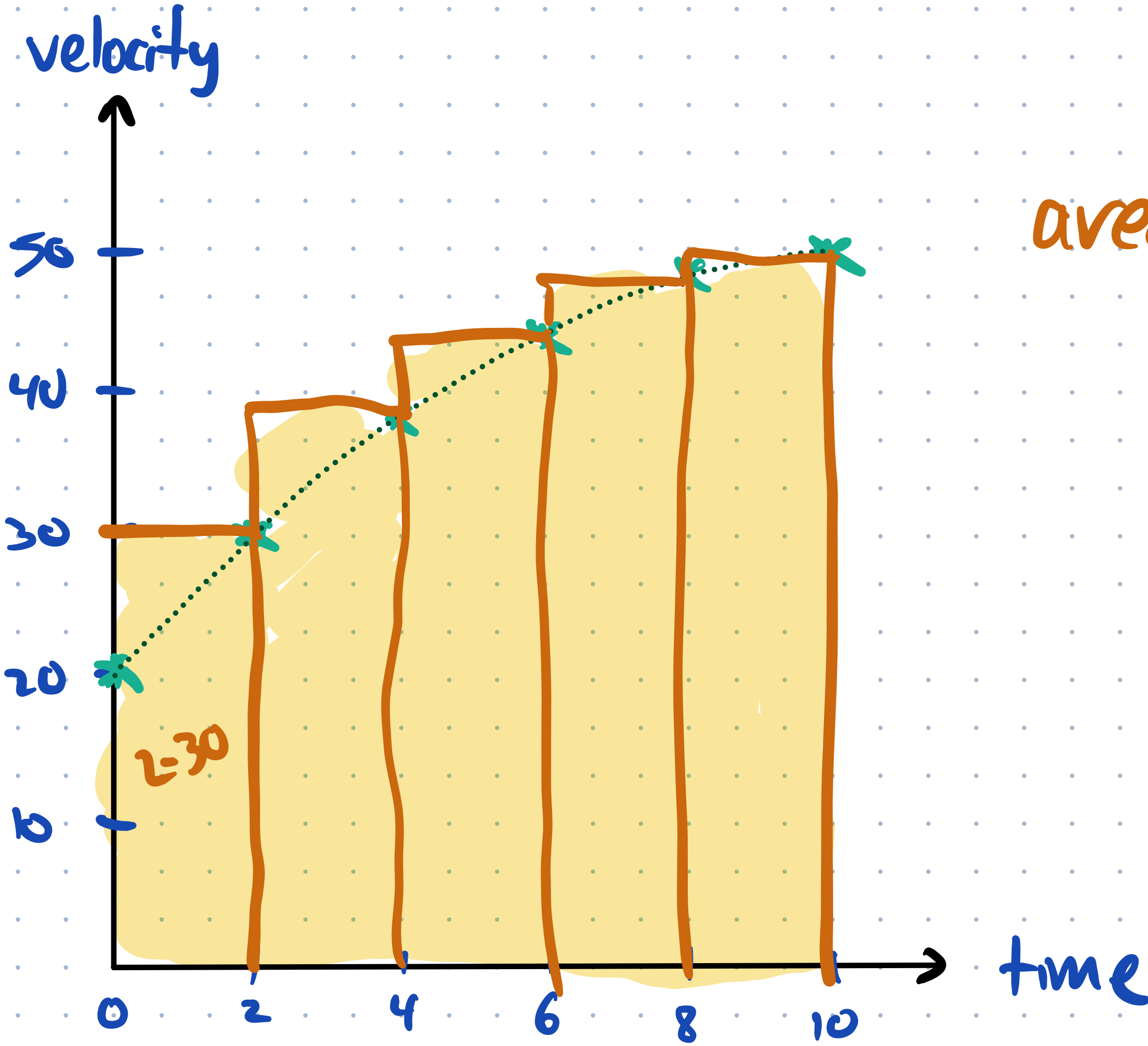
underestimate: $\underbrace{2\cdot 20}_{0\to 2} + \underbrace{2\cdot 30}_{2\to 4} + \underbrace{2\cdot 38}_{4\to 6} + \underbrace{2\cdot 44}_{6\to 8} + \underbrace{2\cdot 48}_{8\to 10} = 360$



Area of is 360 and is our under-estimate

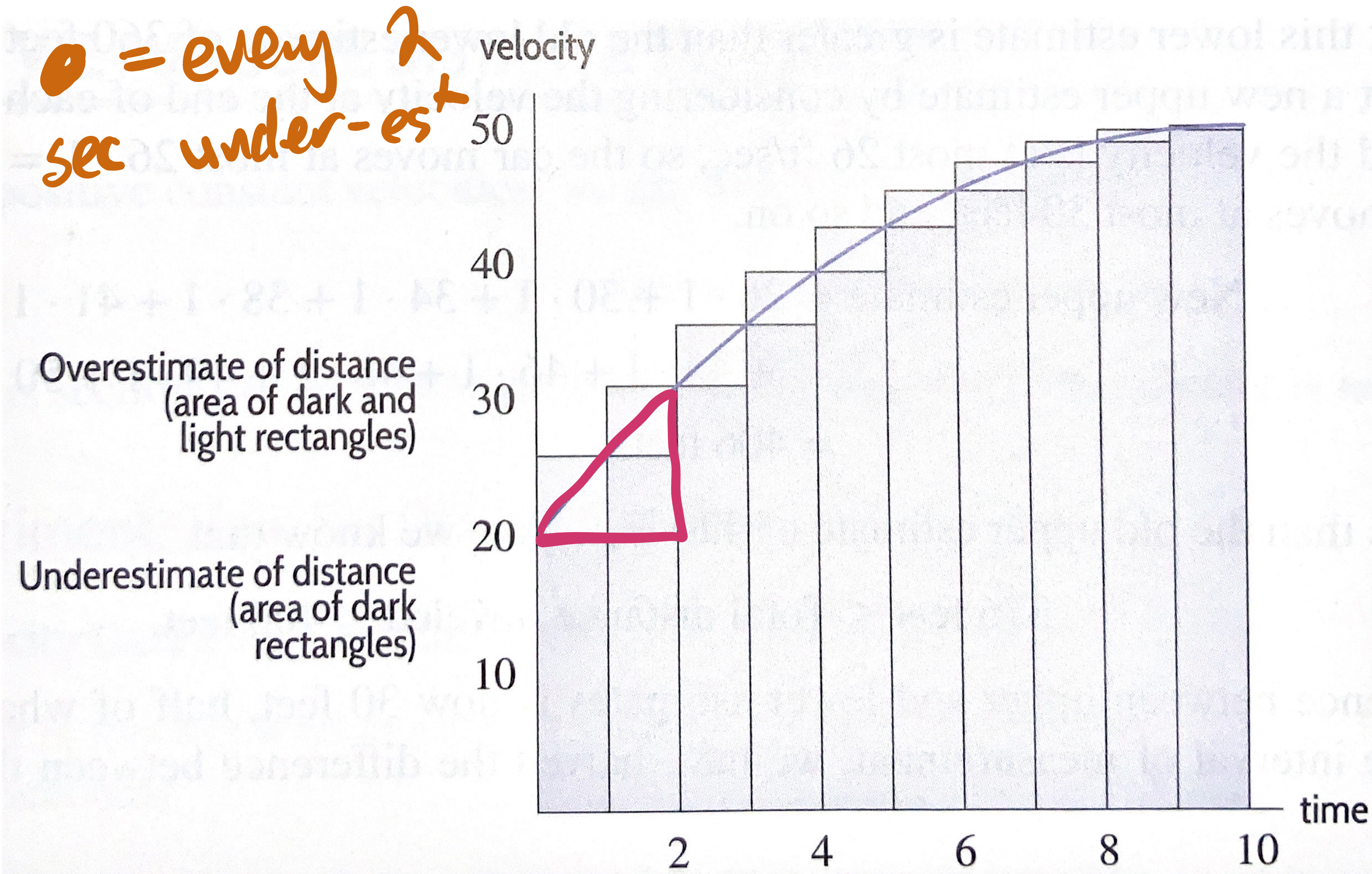
time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Overestimate: $2\cdot 30 + 2\cdot 38 + 2\cdot 44 + 2\cdot 48 + 2\cdot 50 = 420$



area of = 420

over-estimate



From textbook

Readings every 1 second instead of 2

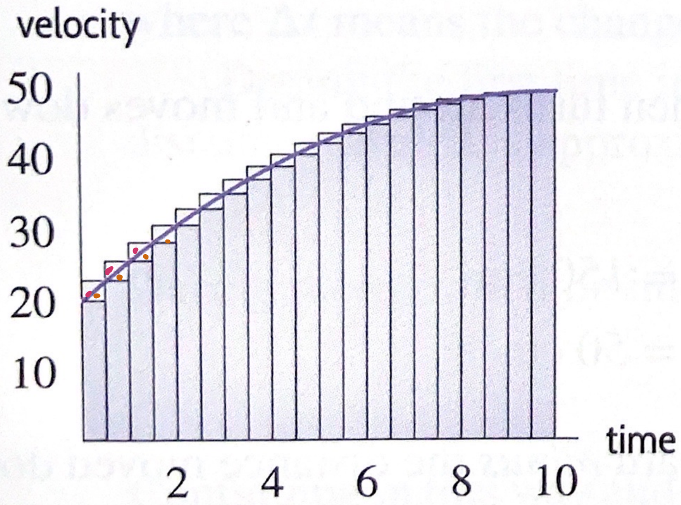


Figure 5.3: Velocity measured every 1/2 second

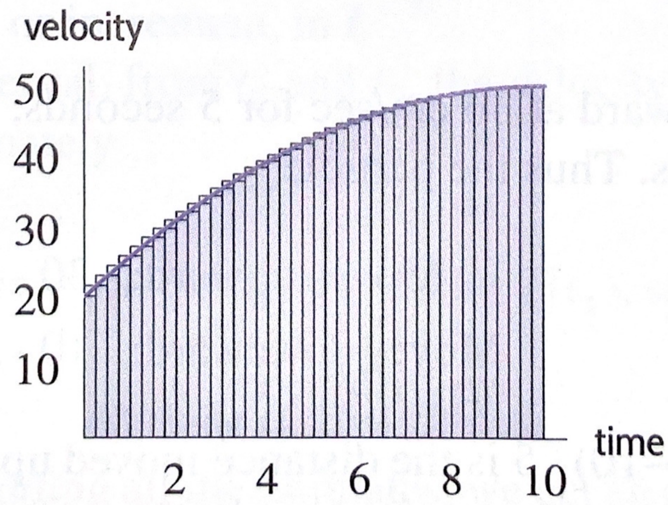


Figure 5.4: Velocity measured every 1/4 second

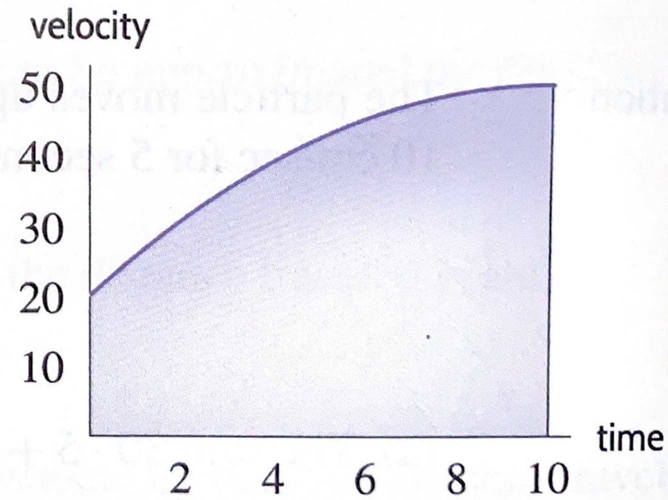


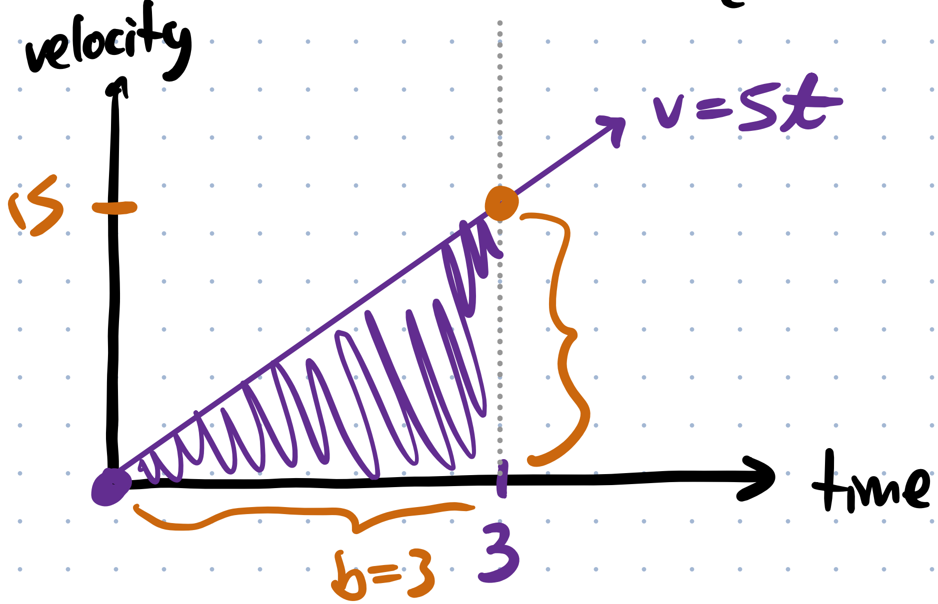
Figure 5.5: Distance traveled is area under curve

Distance travelled is "area under the curve"

(kind of)

Example

The velocity of a bicycle in feet/sec is given by $v(t) = 5t$. How far does the bicycle travel in 3 seconds? ($t=0$ to $t=3$)



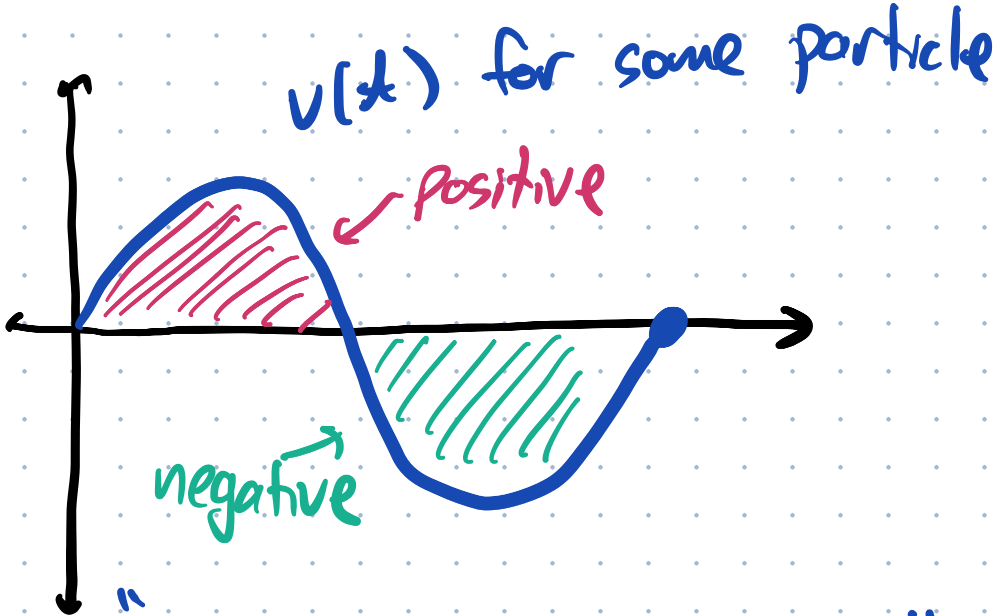
The answer to this Q is = the area between the t-axis and the graph, between $t=0$ and $t=3$.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$= \frac{1}{2} \cdot 3 \cdot 15 = \frac{45}{2} = \boxed{22.5 \text{ ft}}$$

Positive and Negative Velocity

Velocity is speed and direction (positive or negative)



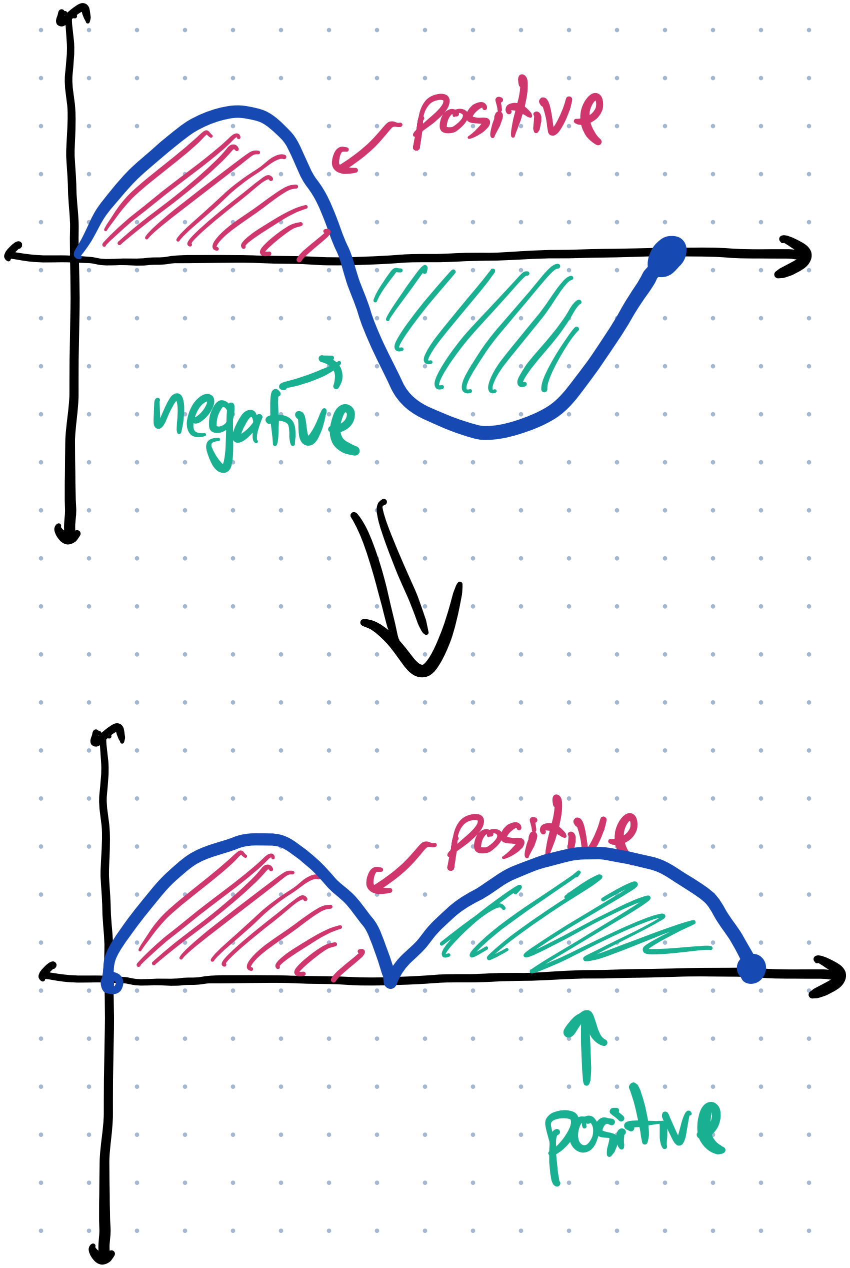
car: 1^{st} half: stopped
speed forward
then stop

2nd half: start reversing
then stop

"Area under the curve" is positive when the curve is above the axis, and negative when below.

"signed area"

This really measures "change in position"
[end position] - [start position]



If you really want "total distance travelled" use the absolute value

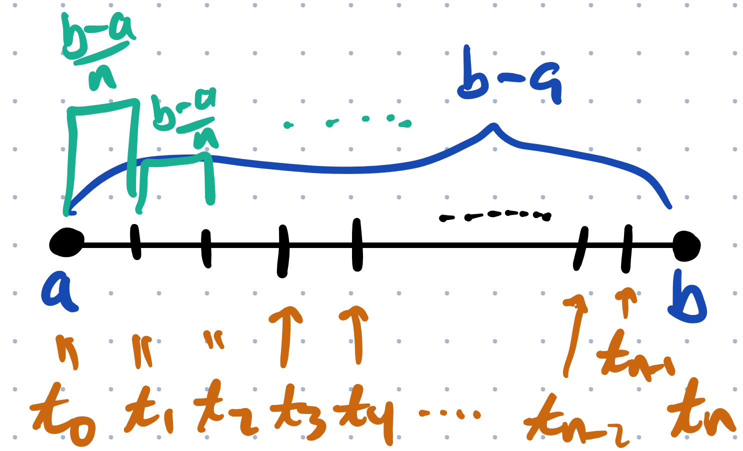
$|v(t)|$

Left and Right Sums

We can estimate the area under a curve by adding up the area of a bunch of rectangles.

Suppose we start at time $t=a$ and end at $t=b$. Suppose we want to use n rectangles.

What is the width of each rectangle?



$$\boxed{\frac{b-a}{n}}$$

(total interval)

← # of rectangles

Previous example: $a=0$, $b=10$, $n=5$

$t_0=0$ $\quad$ $t_3=6$

$t_1=2$ $\quad$ $t_4=8$

$t_2=4$ $\quad$ $t_5=10$

For the height of each rectangle we can either

(1) always use the value of the function at the left endpoint

(2) right endpoint

(3) midpoint

n rectangles

width of each rectangle:

$\frac{b-a}{n} = \boxed{\Delta t}$

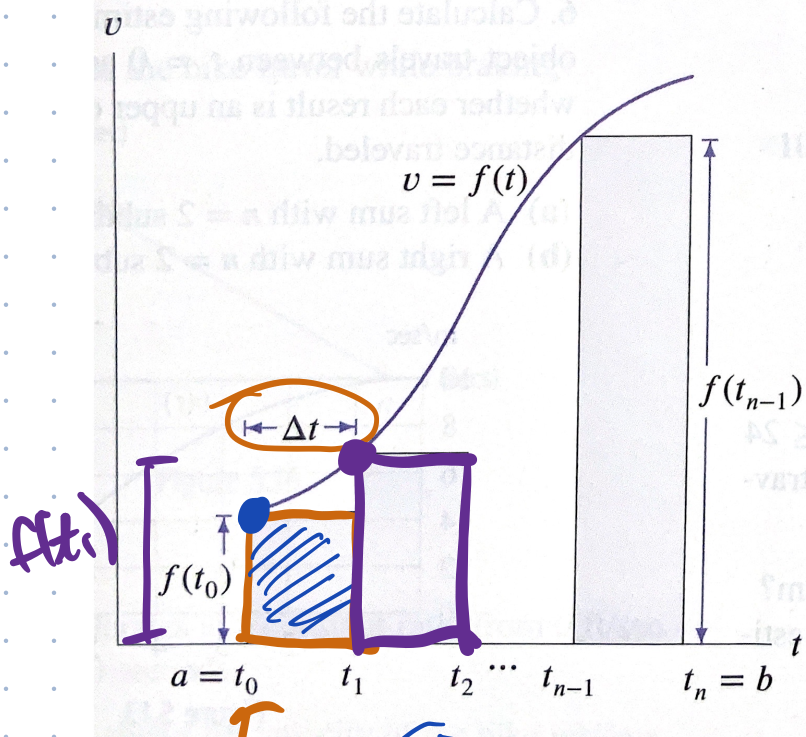


Figure 5.8: Left-hand sums

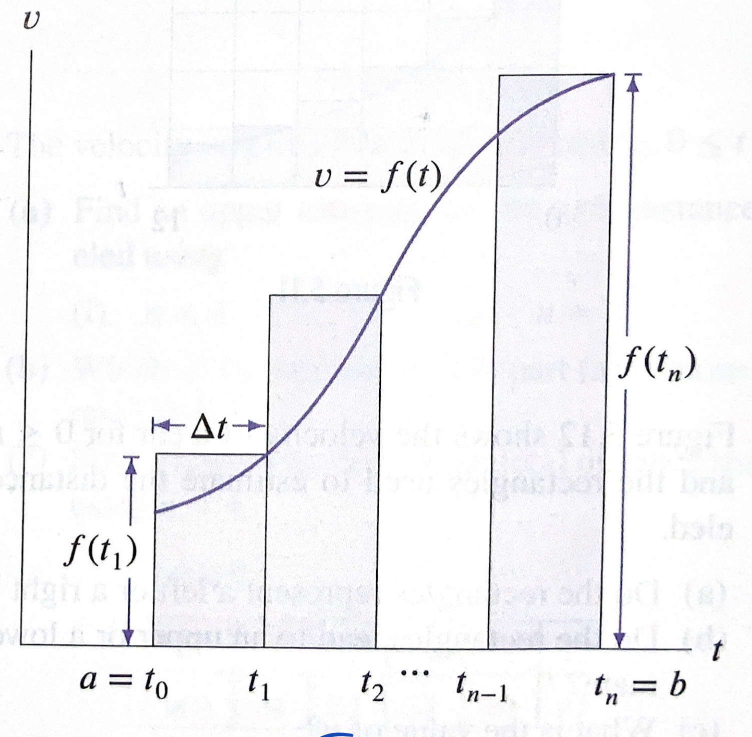
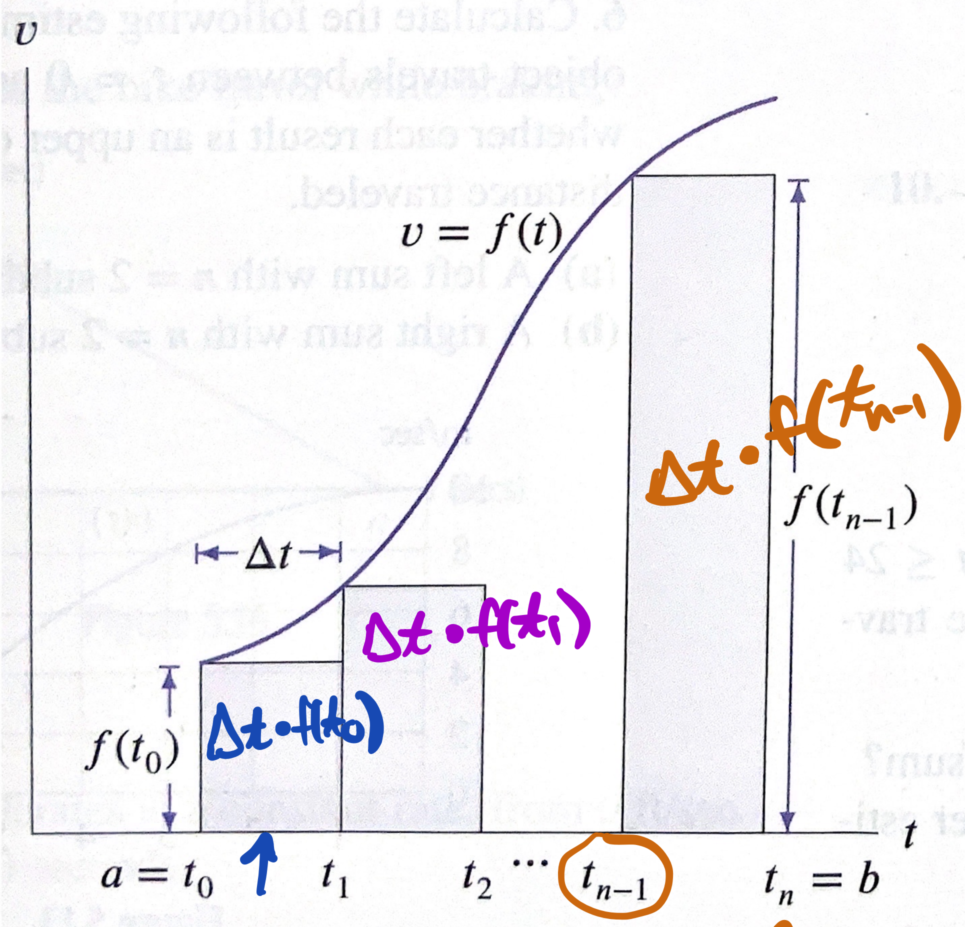


Figure 5.9: Right-hand sums



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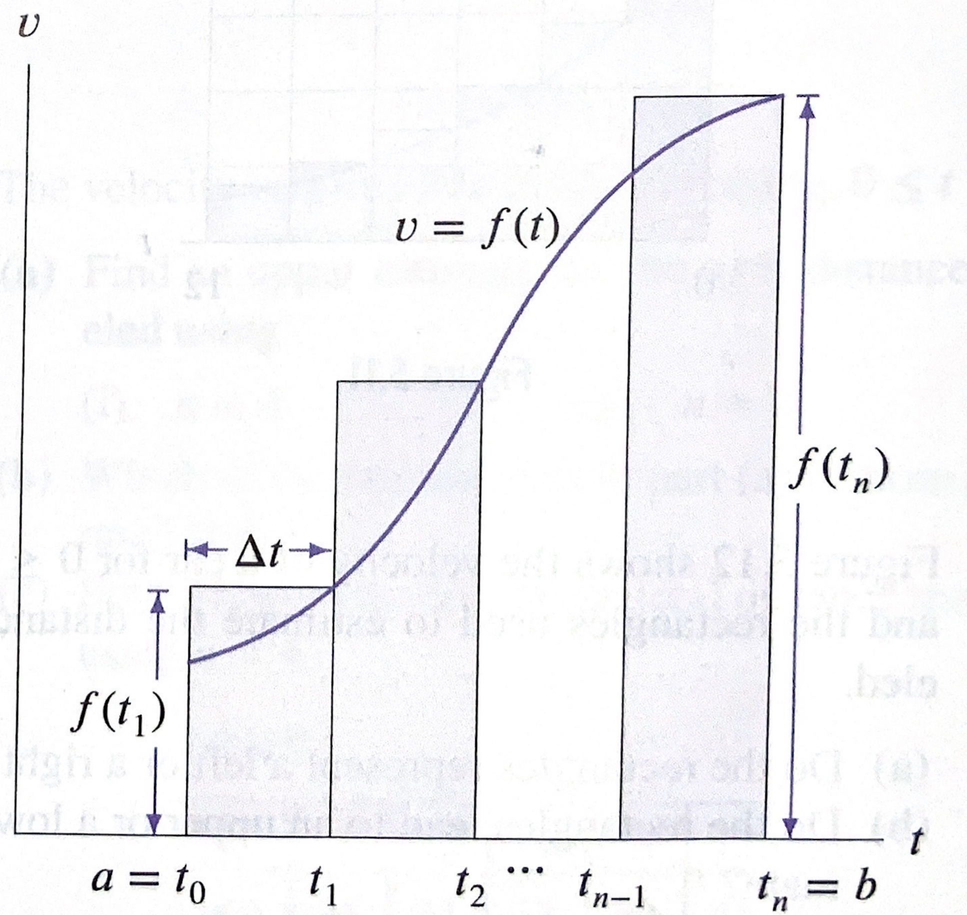
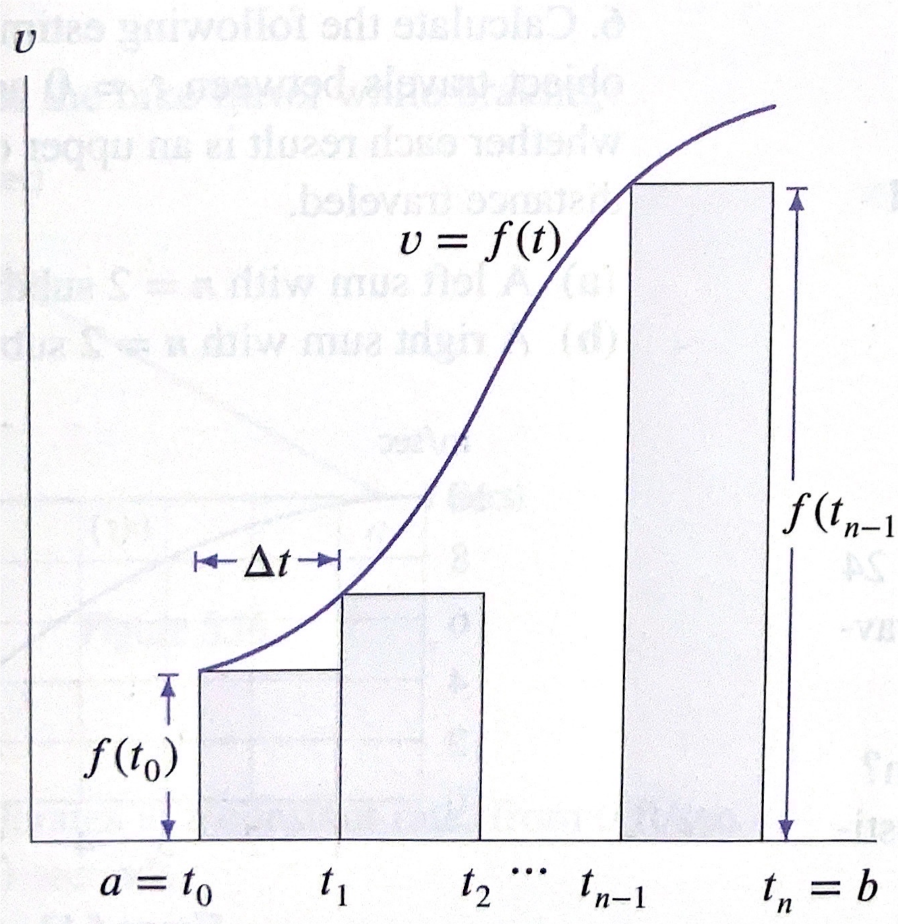


Figure 5.9: Right-hand sums

Left sum

$$\text{Area}: [\Delta t \cdot f(t_0)] + [\Delta t \cdot f(t_1)] + \cdots + [\Delta t \cdot f(t_{n-1})]$$

$$= \Delta t \cdot (f(t_0) + f(t_1) + \cdots + f(t_{n-1}))$$



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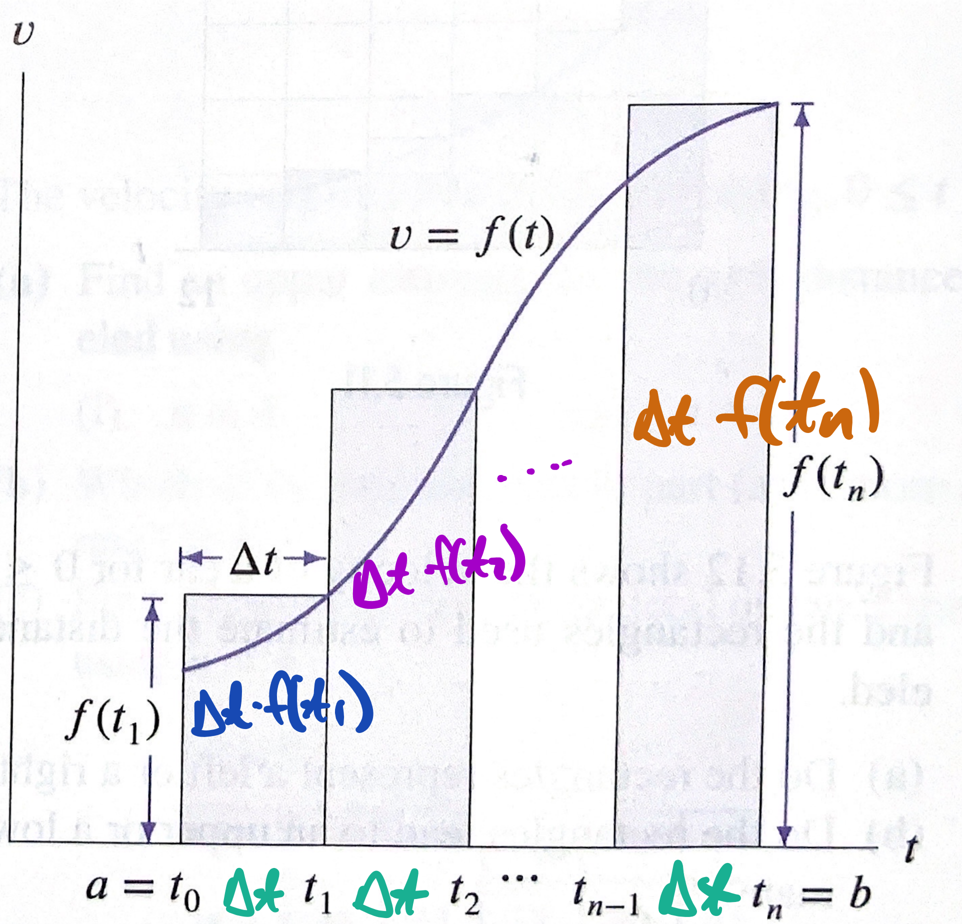


Figure 5.9: Right-hand sums

Right sum:

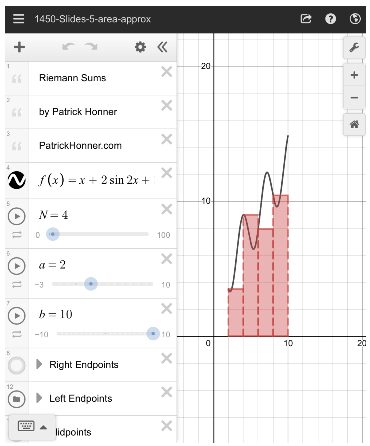
Area: $\Delta t \cdot f(t_1) + \Delta t \cdot f(t_2) + \cdots + \Delta t \cdot f(t_n)$

$= \Delta t \cdot \left(f(t_1) + f(t_2) + \cdots + f(t_n) \right)$

$$\text{Left} = \Delta t \cdot \left(f(t_0) + f(t_1) + \cdots + f(t_{n-1}) \right)$$

$$\text{Right} = \Delta t \cdot \left(f(t_1) + f(t_2) + \cdots + f(t_n) \right)$$

$$\text{Right} - \text{Left} = \Delta t \cdot \left(f(t_n) - f(t_0) \right)$$



Section 5.2: The Definite Integral

opposite of a derivative

"Σ" capital greek sigma σ

Let "$P(i)$" be some mathematical expression in terms of the variable i.

$$\sum_{i=a}^{i=b} P(i) = P(a) + P(a+1) + P(a+2) + \ldots + P(b-1) + P(b)$$

"add $P(i)$ for all values of i (whole #s) between a and b"

$$\sum_{i=3}^{6} i^2 = 3^2 + 4^2 + 5^2 + 6^2 = 86$$

$$\text{Left} = \Delta t \cdot \left(f(t_0) + f(t_1) + \cdots + f(t_{n-1}) \right)$$

$$= \sum_{i=0}^{n-1} (\Delta t) \cdot f(t_i)$$

$$= (\Delta t) \cdot \sum_{i=0}^{n-1} f(t_i)$$

$$\text{Right} = \Delta t \cdot \left(f(t_1) + f(t_2) + \cdots + f(t_n) \right)$$

$$= \sum_{i=1}^{n} (\Delta t) \cdot f(t_i)$$

$$= (\Delta t) \sum_{i=1}^{n} f(t_i)$$