Math 1450 - Calculus 1

Wed, Nov 19

Announcements:

* Thursday:
  - → Quiz 9 (last one!)
  - → Homework 12

  } 4.7 + 4.8

* Next Week: lecture Monday
  discussion Tuesday
  nothing Wed, Thurs, Fri

* Final Exam:
  Wednesday, Dec 10, 8pm - 10pm
  Weaster Auditorium

Today:

→ 4.8: Parametric Equations

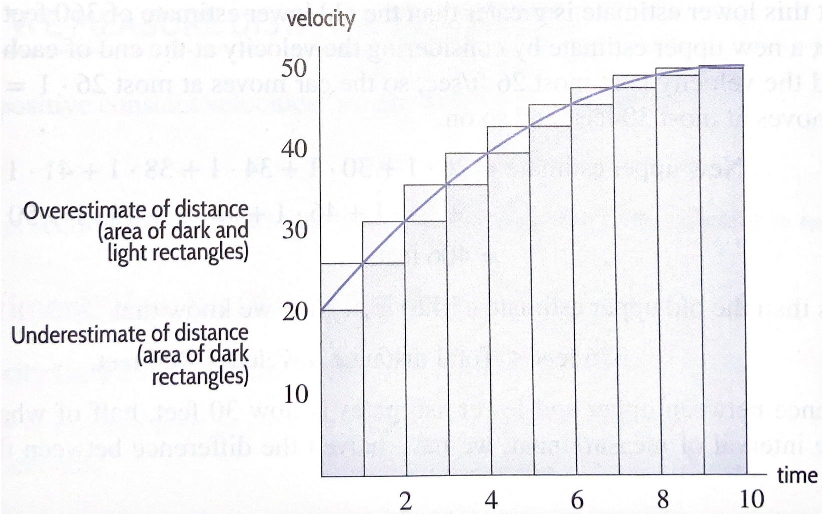
→ 5.1: How do we measure distance traveled?

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1



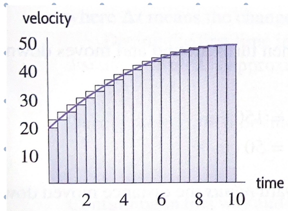


Figure 5.3: Velocity measured every 1/2 second

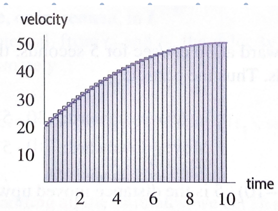


Figure 5.4: Velocity measured every 1/4 second

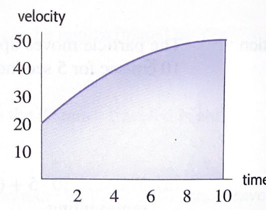


Figure 5.5: Distance traveled is area under curve

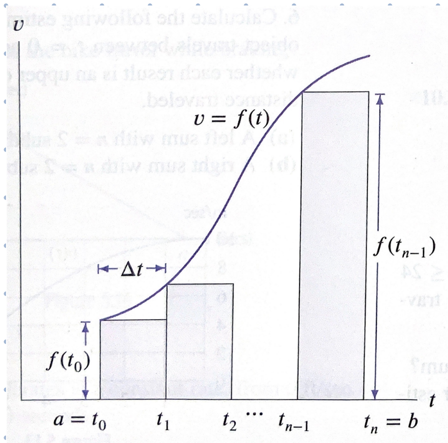


Figure 5.8: Left-hand sums

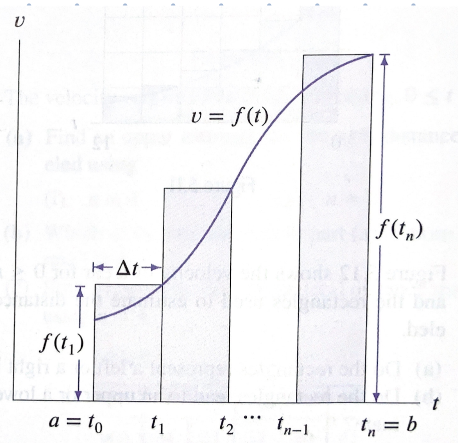


Figure 5.9: Right-hand sums

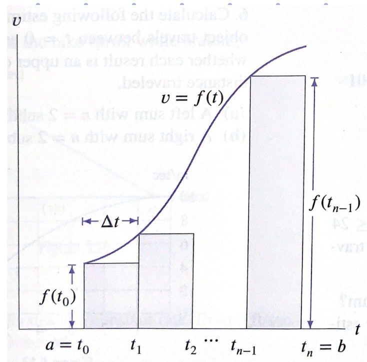


Figure 5.8: Left-hand sums

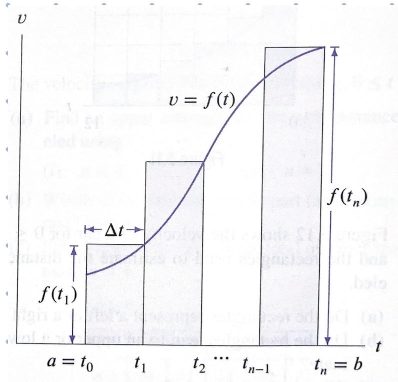
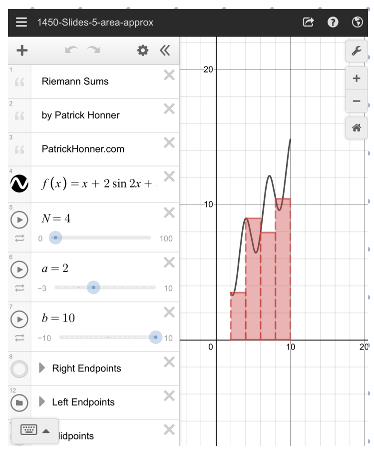


Figure 5.9: Right-hand sums



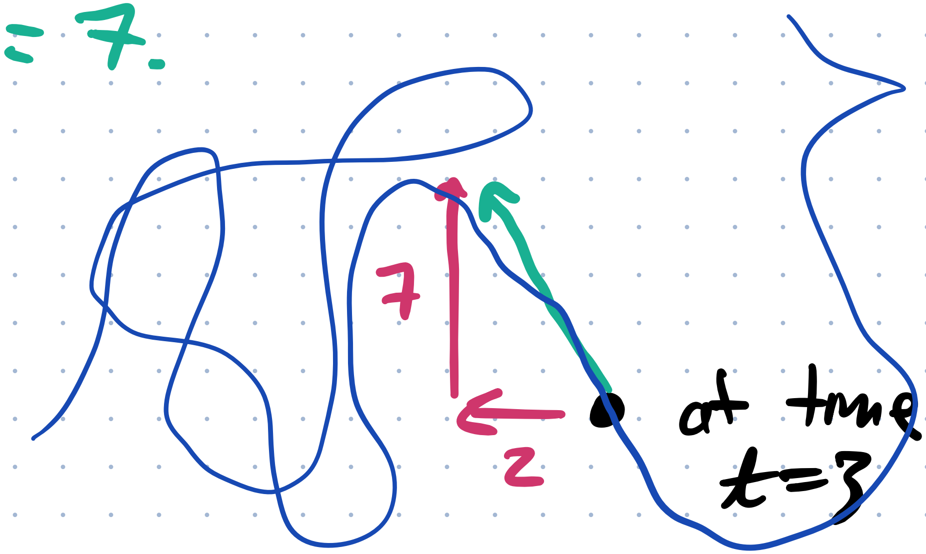
Calculus Stuff for 4.8

How do we find the slope of a curve defined by the parametric equation $(x(t), y(t))$?

(1) Compute $\frac{dx}{dt}$ ("$x'(t)$") and $\frac{dy}{dt}$ ("$y'(t)$")

Pretend that at time $t=3$, we have $x'(3) = -2$ and $y'(3) = 7$.

What does this mean?



slope at this point: $\frac{\text{rise}}{\text{run}} = \frac{7}{-2} = -\frac{7}{2}$

Moral :

The slope of the parametric equation $(x(t), y(t))$ at time t is:

$$\frac{dy}{dx} \qquad \frac{y'(t)}{x'(t)} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{dy/dt}{dx/dt}$$

same thing with two different notations

Ex: What function is traced by the curve $x(t) = \boxed{\sin(3t) - 3}$

$y(t) = -(\sin(3t))^2 + 2$

Let $S = \sin(3t)$

$x(t) = S - 3$

$y(t) = -S^2 + 2$

$y = -S^2 + 2$

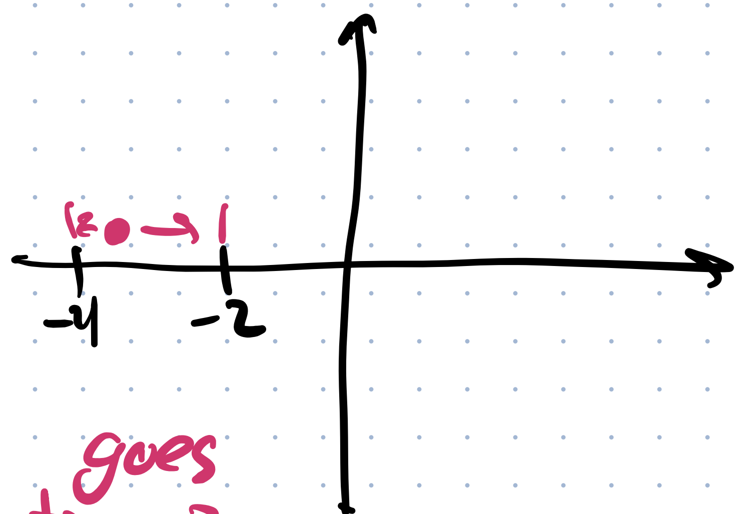
$x(t) + 3 = S$

$y = -(x(t)+3)^2 + 2$

$y = -(x+3)^2 + 2$ where x goes from -4 to -2

$\sin(3t)$ is always between -1 and 1

$\sin(3t) - 3$ is always between -4 and -2



Section 5.1: How do we measure distance traveled?

Suppose you're driving a car, and as you're speeding up, you look down at the spedometer every 2 seconds and write down your speed.

Can you tell how far you traveled?

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Not exactly but we can estimate.

→ Assuming speed is always increasing.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Between time 0 and time 2, you travelled
at least $\left(20 \frac{ft}{sec}\right) \cdot (2 \text{ sec}) = 40$ ft

Between $t=2$ and $t=4$, you travelled
at least $(30 \text{ ft/sec}) \cdot (2 \text{ sec}) = 60$ ft

Overall: $\underbrace{2 \cdot 20}_{0 \to 2} + \underbrace{2 \cdot 30}_{2 \to 4} + \underbrace{2 \cdot 38}_{4 \to 6} + \underbrace{2 \cdot 44}_{6 \to 8} + \underbrace{2 \cdot 48}_{8 \to 10} + \cancel{2 \cdot 50}$

$= 360$ ft

This must be an underestimate because we said you're always speeding up.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

underestimate: $\underbrace{2\cdot 20}_{0\to 2} + \underbrace{2\cdot 30}_{2\to 4} + \underbrace{2\cdot 38}_{4\to 6} + \underbrace{2\cdot 44}_{6\to 8} + \underbrace{2\cdot 48}_{8\to 10} = 360$

Overestimate:

Between time $t=0$ and $t=2$, travelled <u>at most</u> $2\cdot 30 =$ <u>60 ft</u>

$$\underbrace{2\cdot 30}_{0\to 2} + \underbrace{2\cdot 38}_{2\to 4} + \underbrace{2\cdot 44}_{4\to 6} + \underbrace{2\cdot 48}_{6\to 8} + \underbrace{2\cdot 50}_{8\to 10} = 420$$

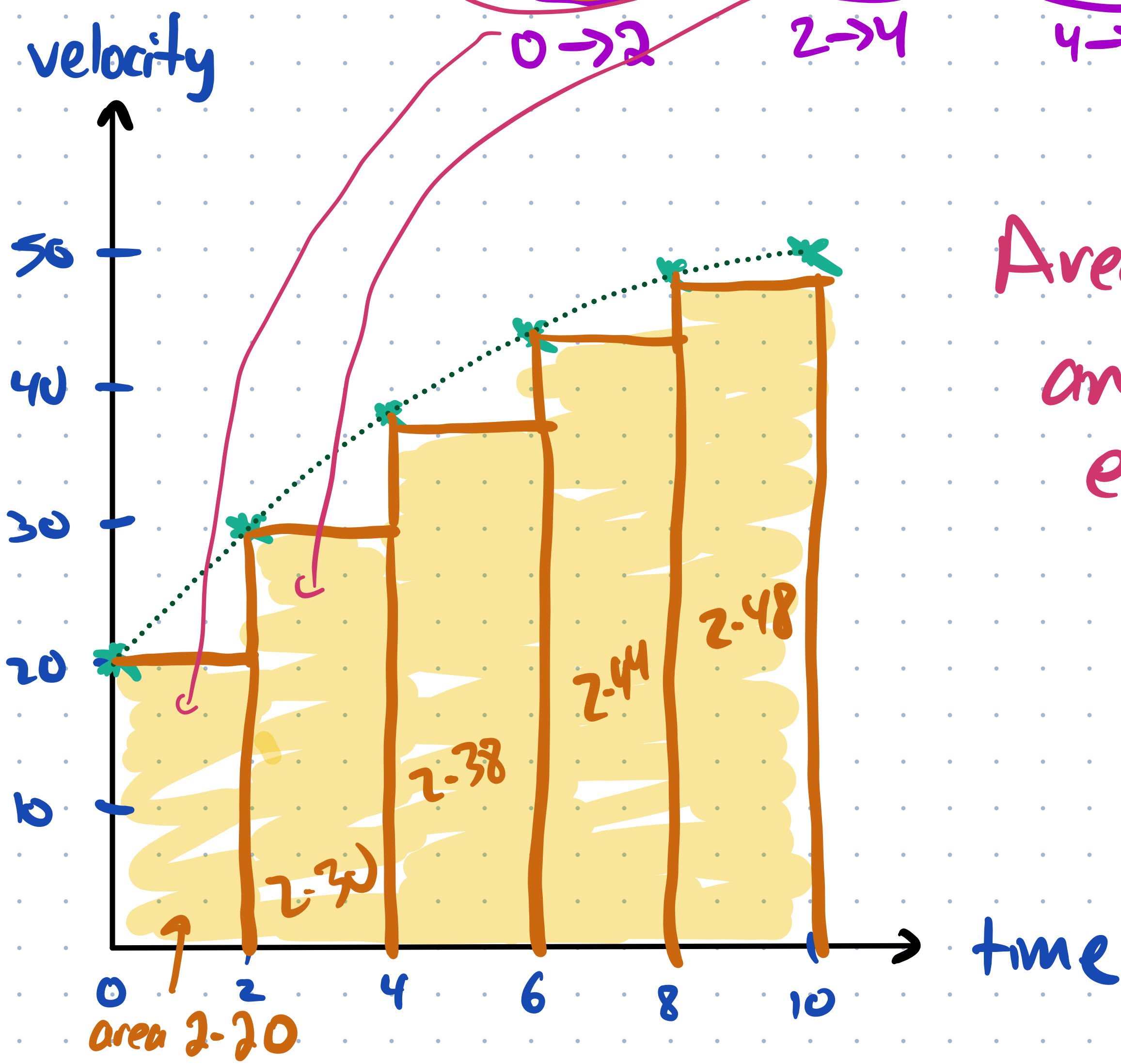
At most 420 ft.

What if we had data every 1 second?

$\rightsquigarrow$ More accurate estimates

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

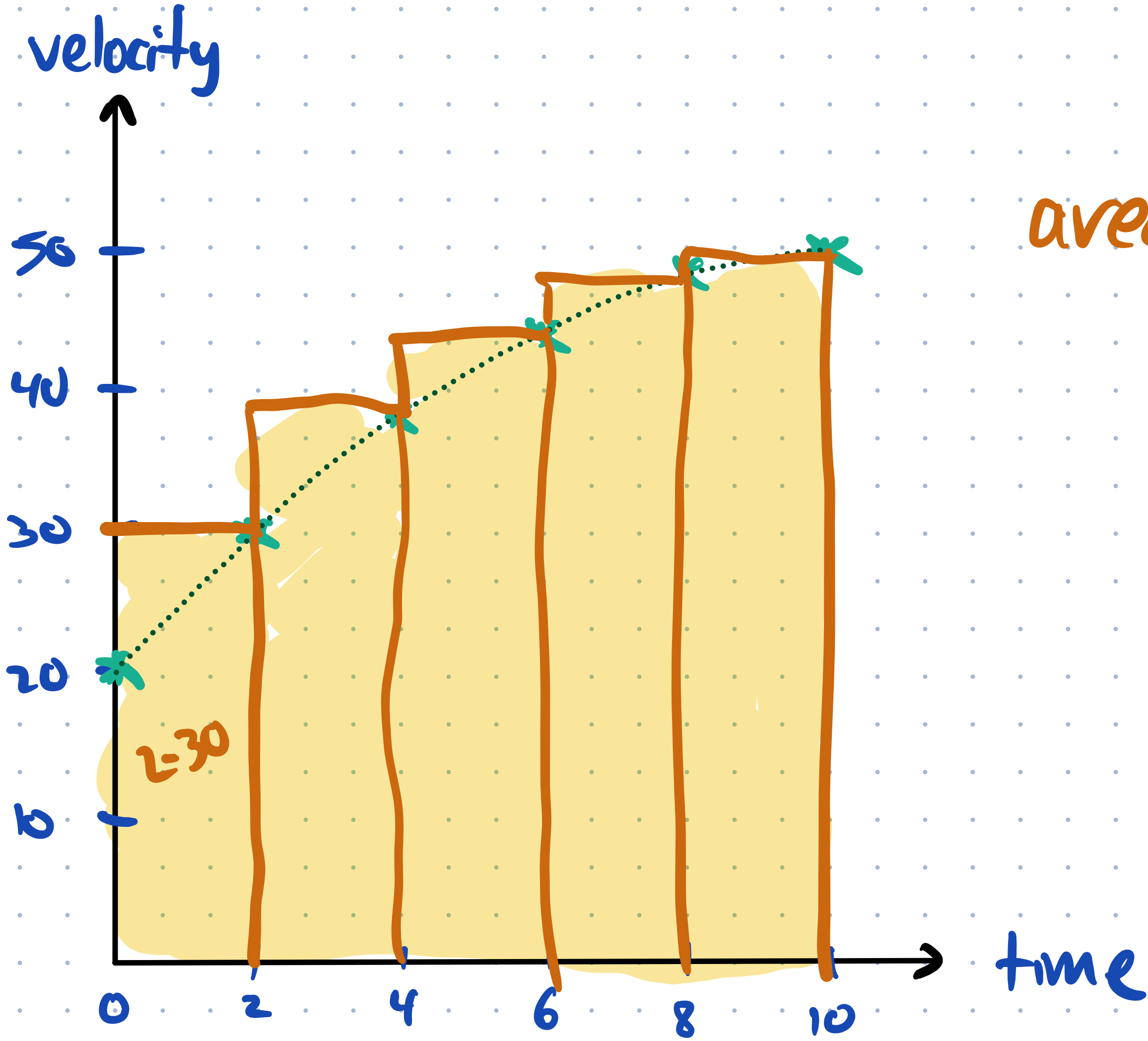
underestimate: $\underbrace{2\cdot 20}_{0\to 2} + \underbrace{2\cdot 30}_{2\to 4} + \underbrace{2\cdot 38}_{4\to 6} + \underbrace{2\cdot 44}_{6\to 8} + \underbrace{2\cdot 48}_{8\to 10} = 360$



Area of is 360 and is our under-estmate

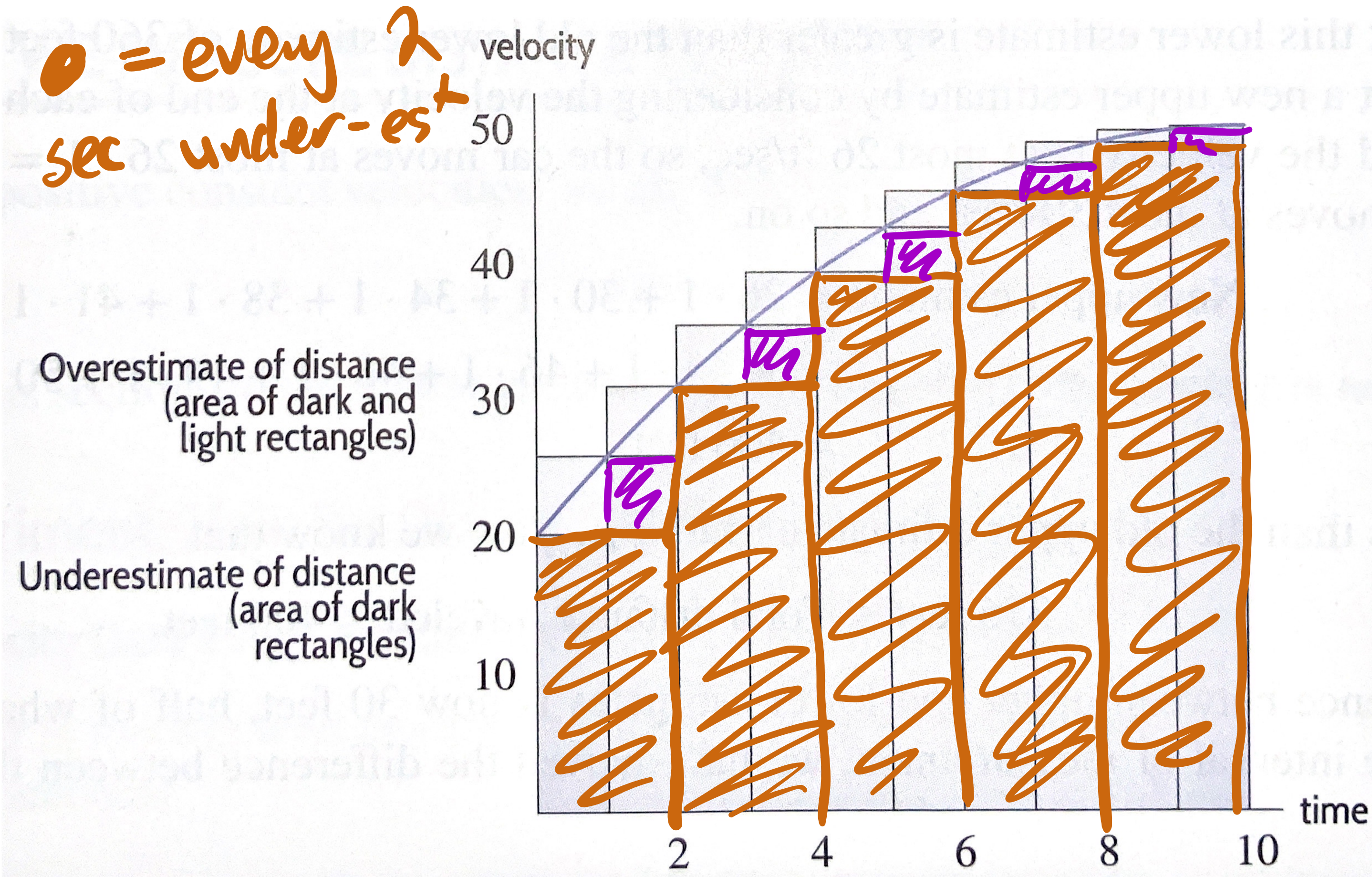
time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Overestimate: $2 \cdot 30 + 2 \cdot 38 + 2 \cdot 44 + 2 \cdot 48 + 2 \cdot 50 = 420$



area of = 420

over-estimate



From textbook

Readings every 1 second instead of 2

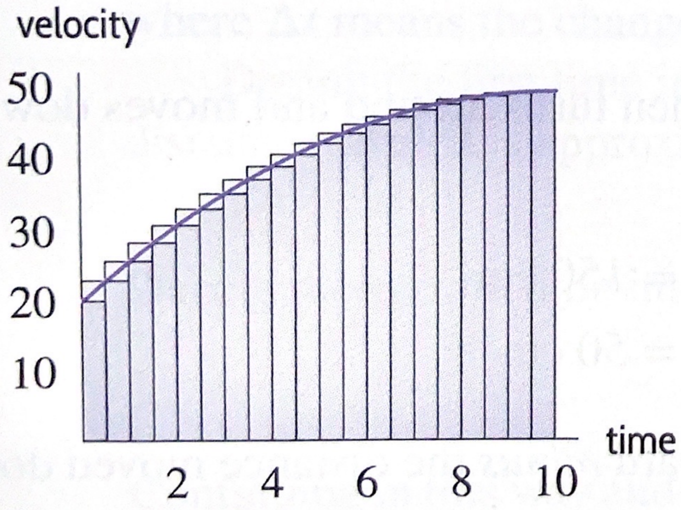


Figure 5.3: Velocity measured every 1/2 second

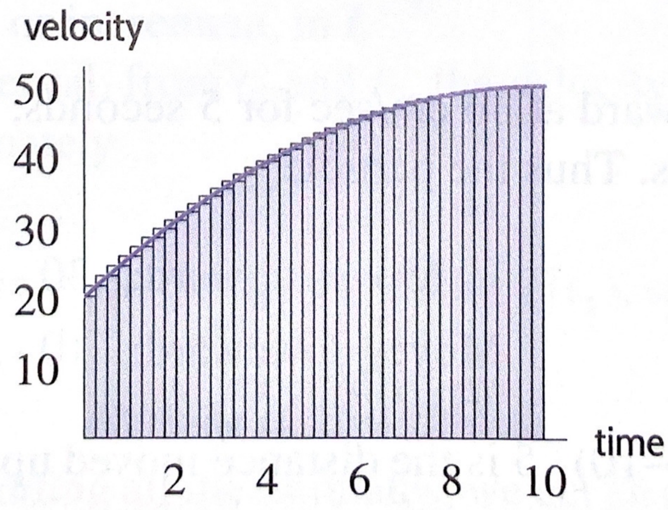


Figure 5.4: Velocity measured every 1/4 second

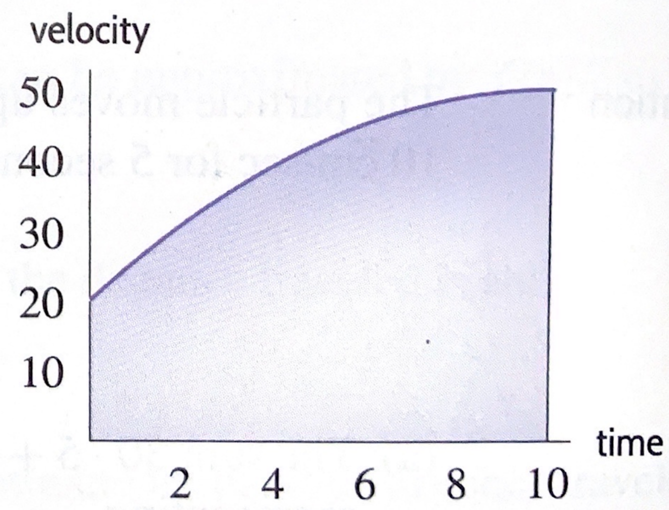


Figure 5.5: Distance traveled is area under curve

Distance travelled is "area under the curve" (kind of)