Math 1450 - Calculus 1

Mon, Nov 17

Announcements:

* Thursday:
  → Quiz 9 (last one!)
  → Homework 12
  } 4.7 + 4.8
* Next Week: lecture Monday
  discussion Tuesday
  nothing Wed, Thurs, Fri
* Final Exam:
  Wednesday, Dec 10, 8pm - 10pm
  Weaster Auditorium

Today:

→ 4.8: Parametric Equations

end at 12:50 today

Office Hours

Mondays, 12-1

Wednesdays, 2-3

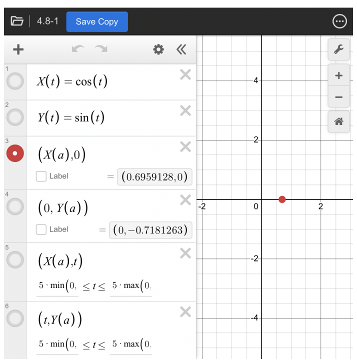
+ Help Desk! 12-1

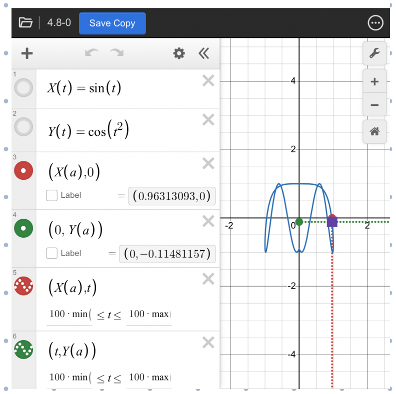
Section 4.8 – Parametric Equations

Parametric Equations are a new way to describe curves in the xy-plane.

Ex: $\underbrace{x = \cos(t) \qquad y = \sin(t)}$

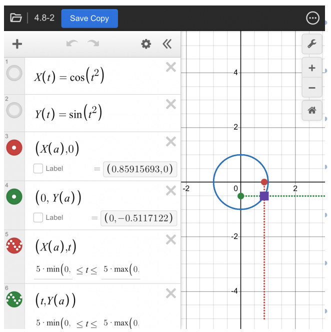
Plots the set of all points of the form $(\cos(t), \sin(t))$, for all possible values of t.





Think about t as "time" and think about parametric equations as plotting the path of a particle over time.

Ex: $x = \cos(t^2)$ $\quad y = \sin(t^2)$



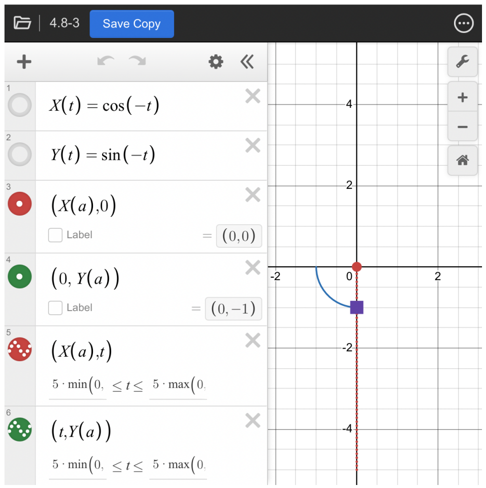
Same circle but traced differently.

As t goes from $-\infty$ to 0, t^2 is going from ∞ to 0 so the circle is traced clockwise

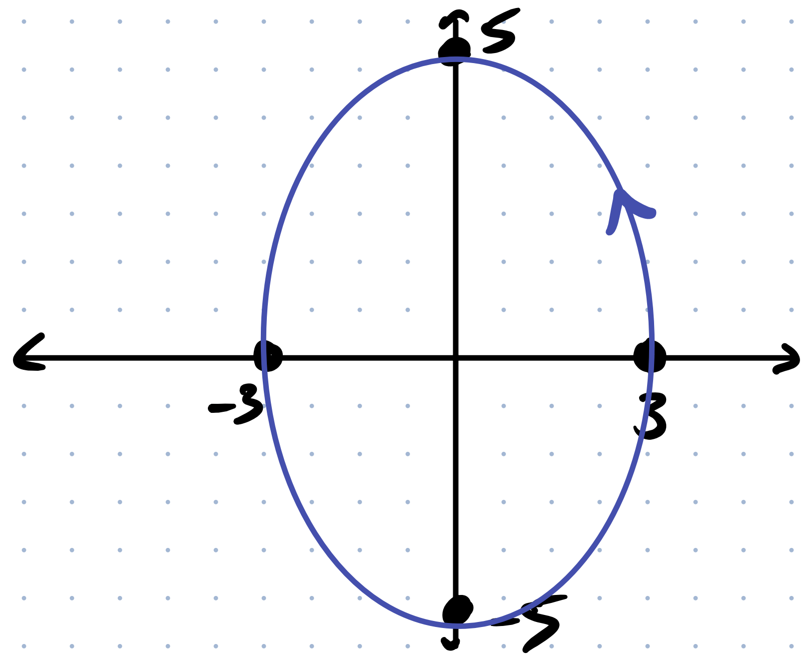
As t goes from 0 to ∞, t^2 also goes 0 to ∞, so the circle is traced CCW .

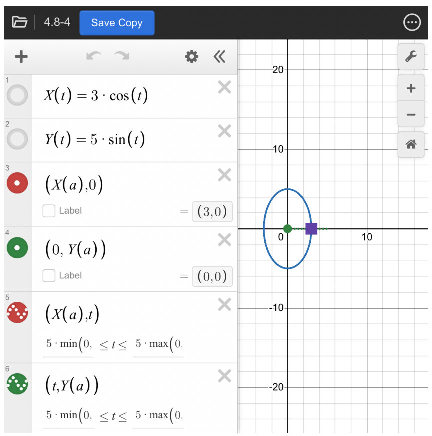
Ex: $x = \cos(-t)$ $y = \sin(-t)$

$\frac{\pi}{2} \leq t \leq \pi$



Ex: $x = 3\cos(t)$ $\quad$ $y = 5\sin(t)$





$t=0: \left(3\cdot\cos(0),\ 5\sin(0)\right)$

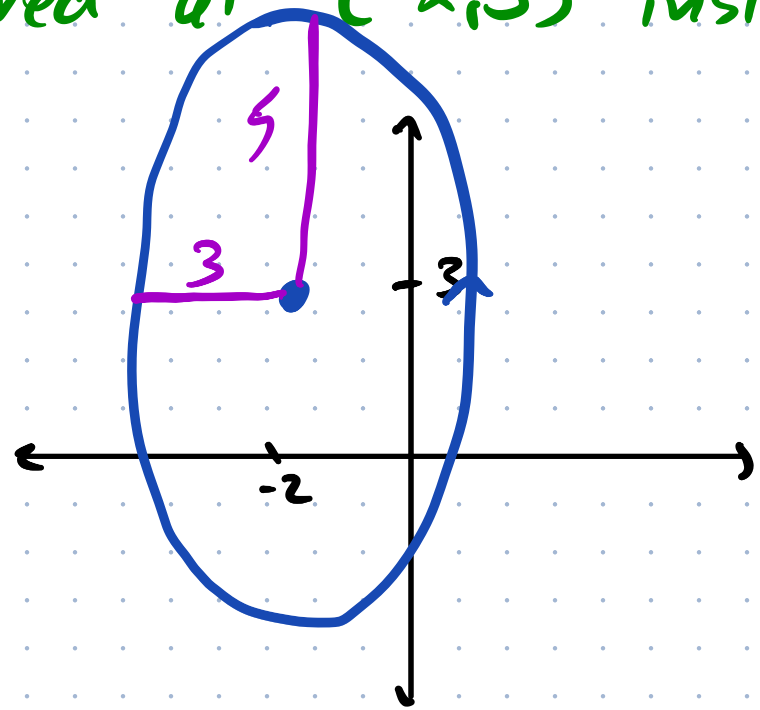
$= (3,0)$

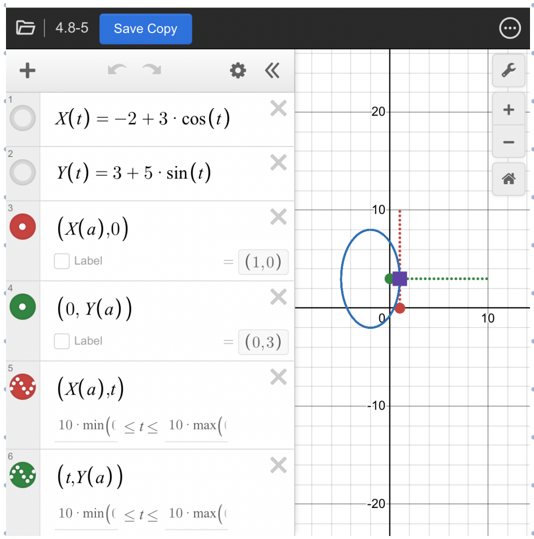
$t=\pi/2: \left(3\cos(\pi/2),\ 5\sin(\pi/2)\right)$

$= (0,5)$

Ex: $x = -2 + 3\cos(t)$ $\quad$ $y = 3 + 5\sin(t)$

same ellipse, but centered at $(-2,3)$ instead of $(0,0)$





Ex:

$x(t) = 2t - 1 \qquad y(t) = -4t + 3$

every time t goes up by 1,

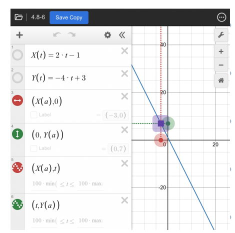
x goes over by 2 to the right

y goes down by 4

→ line with slope $\frac{-4}{2} = \boxed{-2}$

→ to get a point it passes through, plug in any t-value

$(x(0), y(0)) = (-1, 3)$



line with slope -2 that passes through $(-1, 3)$ is $\boxed{y = -2x + 1}$

Ex Find a parametric equation for the line $y = 7x - 3$

$x(t) = t \quad y(t) = 7t - 3$

going from parametric to $y=$ form is easy

Calculus Stuff

How do we find the slope of a curve defined by the parametric equation $(x(t), y(t))$?

compute $\frac{dx}{dt}$ (also called $x'(t)$)

compute $\frac{dy}{dt}$ (also called $y'(t)$)

The slope of $(x(t), y(t))$ at a particular time is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$