Math 1450 - Calculus 1

Mon, Nov. 10

Announcements:

* HW 11 due Tuesday tomorrow, covers 4.3 + 4.6

* Exam 3 on Wednesday, 5pm-6pm
  covers $\boxed{3.5}$, 3.6, 3.7, 3.9, 3.10
  trig
  4.1, 4.2, 4.3, $\boxed{4.6}$ — related rates

→ Exam 3 study guide is on our course website

* Course withdrawal deadline is Fri

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 4.6: Related Rates

→ Review

4.6: Related Rates

This section: Two different quantities are related.

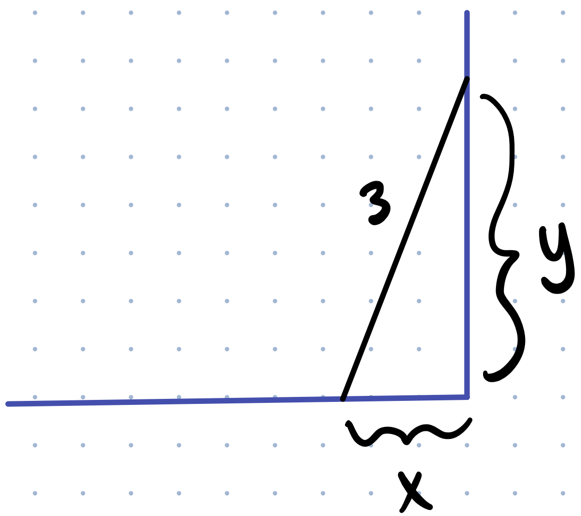
* radius of a sphere vs. volume of a sphere
  $V = \frac{4}{3}\pi \cdot r^3$
* position of a plane in air vs. the angle it's making with some landmark
* speed of a car vs. fuel efficiency

How is the rate of change of one related to the rate of change of the other?

Ex: A 3-meter ladder stands against a high wall. The foot of the ladder moves outward at a constant speed of 0.1 m/s. When the foot is 1m from the wall, how fast is the top of the ladder falling? What about 2m?

(1) Draw a picture

(2) Give variable names to all relevant quantities!



Q: when $x=1$ and $\frac{dx}{dt}=0.1$

what is $\frac{dy}{dt}$?

Need an equation that relates x and y

$$x^2+y^2=9$$

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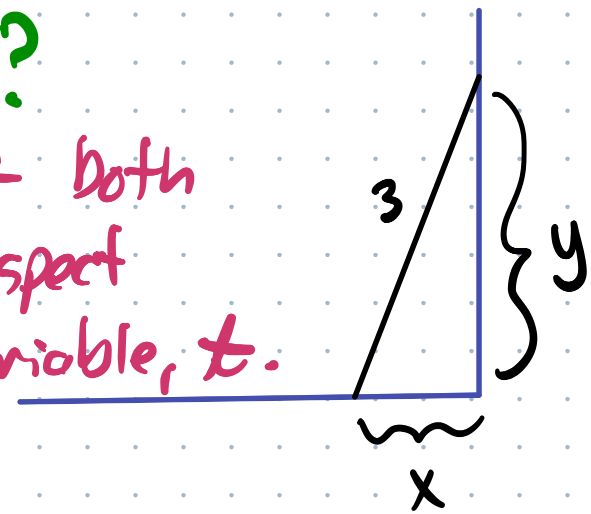

$x^2 + y^2 = 9$

* Take deriv. of both sides with respect to a <u>new</u> variable, t.

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(9)$

$$\boxed{2 \cdot x \cdot \frac{dx}{dt} + 2 \cdot y \cdot \frac{dy}{dt} = 0}$$

chain rule

equation that relates the rates of change of x and y

Know x, $\frac{dx}{dt}$

We can figure out y because $x^2+y^2=9$.

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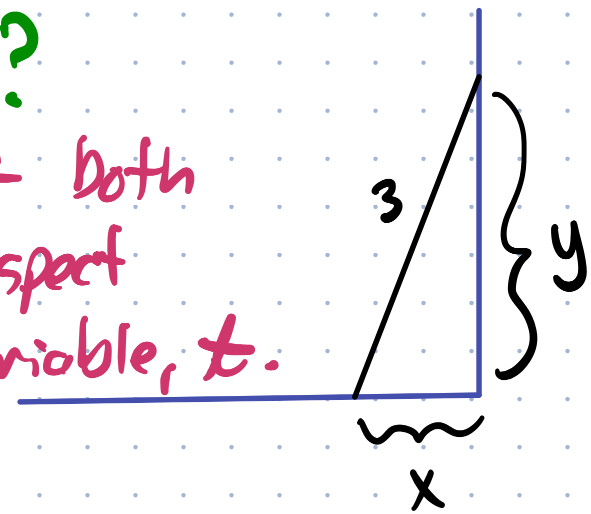

$$x^2+y^2=9$$

* Take deriv. of both sides with respect to a new variable, t.

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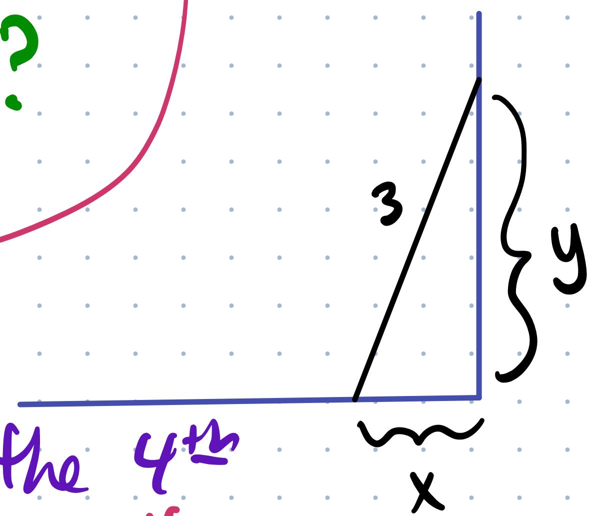

$$2\cdot x\cdot\frac{dx}{dt}+2\cdot y\cdot\frac{dy}{dt}=0$$

Four variables, x, $\frac{dx}{dt}$, y, $\frac{dy}{dt}$, so if we have any 3, we can solve for the 4th

When the foot is 1 m away from the wall:

$x=1$

$\frac{dx}{dt}=0.1$

$y=\sqrt{8}$

$$2\cdot(1)\cdot(0.1)+2\cdot\sqrt{8}\cdot\frac{dy}{dt}=0$$

$$\frac{2\sqrt{8}\cdot\frac{dy}{dt}}{2\sqrt{8}}=\frac{-0.2}{2\sqrt{8}} \Longrightarrow \frac{dy}{dt}=-\frac{0.1}{\sqrt{8}}\approx -0.035 \text{ m/s}$$

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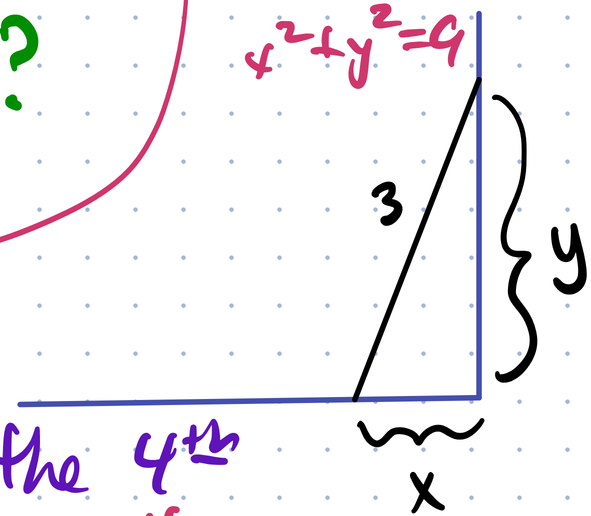


$$2\cdot x\cdot\frac{dx}{dt}+2\cdot y\cdot\frac{dy}{dt}=0$$

Four variables, x, $\frac{dx}{dt}$, y, $\frac{dy}{dt}$, so if we have any 3, we can solve for the 4th

When the foot is 2 m away from the wall:

$x=2$

$\frac{dx}{dt}=0.1$

$y=\sqrt{5}$

$$\frac{dy}{dt}=\frac{-0.4}{2\sqrt{5}}\approx -0.089 \text{ m/s}$$

$2^2+y^2=9$

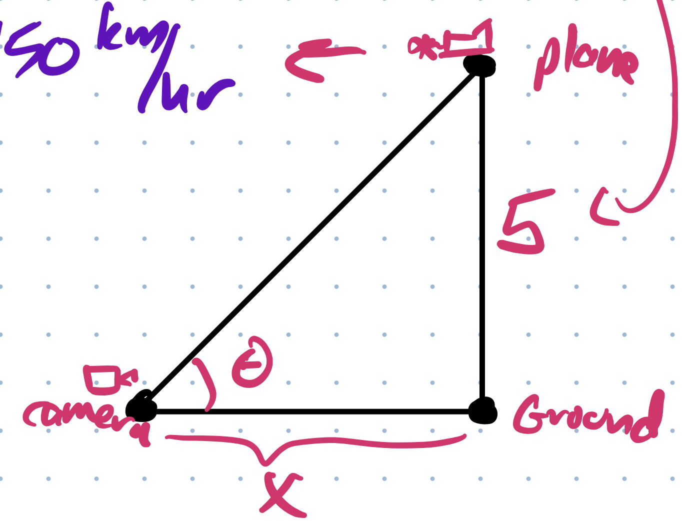
$y=\sqrt{5}$

What about $x=2.999$?

Ex: An airplane flying at 450 km/hr at a constant altitude of 5km is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed. When $\theta = \pi/3$ how fast does the camera have to rotate to keep the plane in view?

$$\frac{dx}{dt} = -450 \text{ km/hr}$$

$$\frac{d\theta}{dt}$$



Need an equation relating θ and x

$$\boxed{\tan(\theta) = \frac{5}{x}}$$

$$\frac{d}{dt}\left(\tan(\theta)\right) = \frac{d}{dt}\left(\frac{5}{x}\right)$$

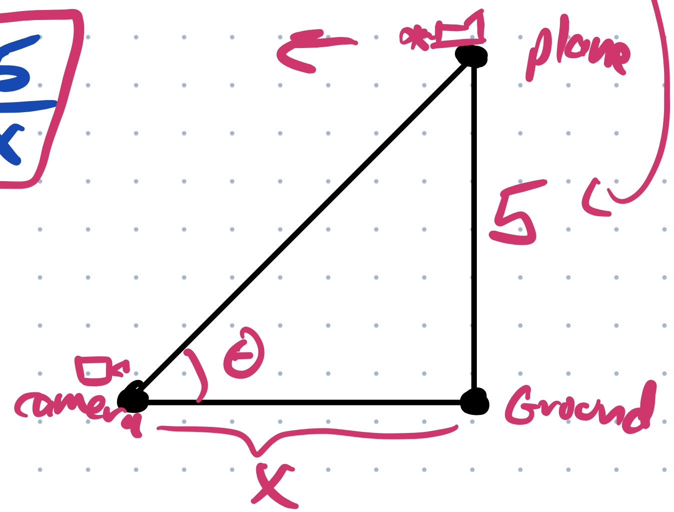
$$\frac{1}{\cos^2(\theta)} \cdot \frac{d\theta}{dt} = -\frac{5}{x^2} \cdot \frac{dx}{dt}$$

chain rule

$$\left(\frac{5}{t}\right)' = \left(5t^{-1}\right)'$$

$$= 5 \cdot (-1)t^{-2}$$

Ex: An airplane flying at $\boxed{450 \text{ km/hr}}$ at a constant altitude of $\boxed{5\text{km}}$ is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed. When $\theta = \pi/3$ how fast does the camera have to rotate to keep the plane in view?



$$\boxed{\tan(\theta) = \frac{5}{x}}$$

$$\frac{1}{\cos^2(\theta)} \cdot \frac{d\theta}{dt} = -\frac{5}{x^2} \cdot \frac{dx}{dt}$$

Know: $\frac{dx}{dt} = -450$, $\boxed{\theta = \pi/3}$, $x = \frac{5}{\sqrt{3}}$

Want to find $\frac{d\theta}{dt}$

$$\frac{1}{\cos^2\left(\frac{\pi}{3}\right)} \cdot \frac{d\theta}{dt} = \frac{-5}{\left(5/\sqrt{3}\right)^2} \cdot (-450)$$

"$\frac{1}{4}$"

$$\Rightarrow 4 \cdot \frac{d\theta}{dt} = -\frac{5}{25/3}(-450)$$

$$\frac{d\theta}{dt} = \frac{1}{4} \cdot \frac{-5}{25/3} \cdot (-450)$$

$$\boxed{= 67.5 \text{ rad/hr}}$$

(≈ 1 degree/sec)