Math 1450 - Calculus 1

Wed, Nov. 5

Announcements:

* HW 10 due tomorrow, covers 4.1 and 4.2
* Quiz 8 tomorrow, covers all 4.1+4.2 sugg. HW
* HW 11 due Tuesday, covers 4.3+4.6, big sections!
* Exam 3 on Wednesday, Nov. 12 - 5pm-6pm
  covers 3.5, 3.6, 3.7, 3.9, 3.10
  4.1, 4.2, 4.3, 4.6

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

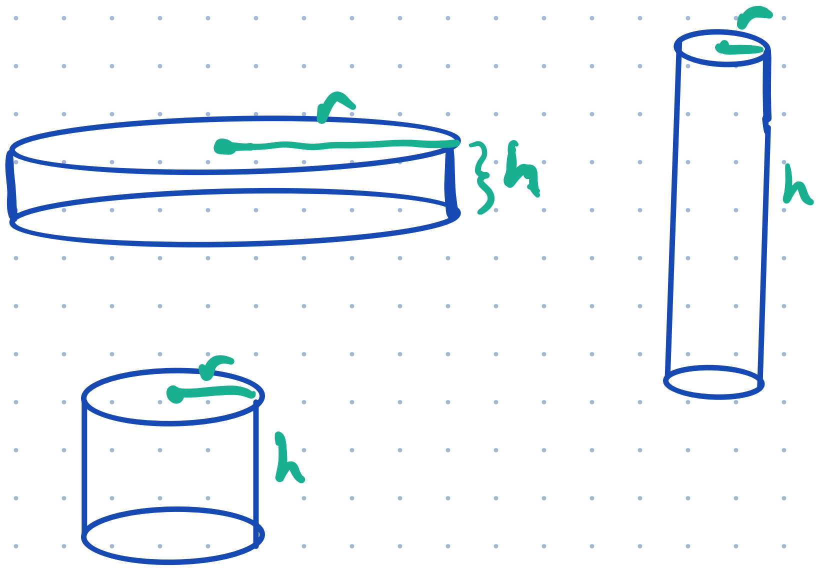
→ 4.3: Optimization + Modeling

→ 4.6: Related Rates

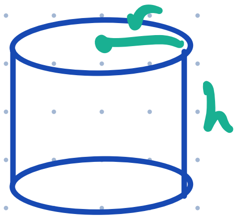
Ex: (from book) What are the dimensions of an aluminum can (cylinder) that holds 40 in^3 of juice and uses the least amount of material?

volume constraint

surface area
quantity we want to minimize



$\boxed{\text{Constraint: Volume} = 40 \quad \text{Goal: minimize surface area}}$



Volume $= \boxed{\pi r^2 \cdot h}$ area of a circle $= \pi r^2$

Surface area $= \underbrace{2\pi r^2}_{\text{top + bottom}} + \underbrace{h \cdot (2\pi r)}_{\text{outside}}$

Two variables, r and h

The constraint tells us how r and h must be related.

$$\boxed{\pi r^2 h = 40}$$

* Use the constraint to solve for one variable in terms of the other.

solve for h

$$h = \frac{40}{\pi r^2}$$

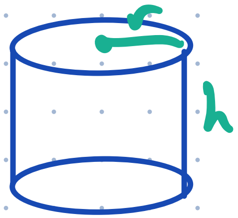
solve for r

$$r = \sqrt{\frac{40}{\pi h}}$$

pick the one that looks nicest

Constraint: Volume $= 40$

Goal: minimize surface area



Volume = $\boxed{\pi r^2 \cdot h}$

Surface area = $\underbrace{2\pi r^2}_{\text{top + bottom}} + \underbrace{h \cdot (2\pi r)}_{\text{outside}}$

$h = \frac{40}{\pi r^2}$

plug in $h = \frac{40}{\pi r^2}$ so we only have one variable

Surface area:

$$2\pi r^2 + 2\pi r \cdot h = 2\pi r^2 + 2\cancel{\pi r} \cdot \left(\frac{40}{\cancel{\pi} r^{\cancel{2}}}\right)$$

$$\boxed{S(r) = 2\pi r^2 + \frac{80}{r}}$$

($r > 0$ because of the real world problem)

Use the stuff from 4.2 to find the global minimum of $S(r)$.

$$S(r) = 2\pi r^2 + \frac{80}{r} \qquad 80 \cdot r^{-1}$$

$$0 < r < \infty$$

$$S'(r) = 4\pi r + 80\cdot(-1\cdot r^{-2})$$
$$= 4\pi r - \frac{80}{r^2}$$

Find critical points: $S'(r)=0$ or $\underbrace{[S'(r) \text{ undefined and } S(r) \text{ defined}]}_{\text{none}}$

$$4\pi r - \frac{80}{r^2} = 0$$

$$\Rightarrow 4\pi r \cdot r^2 = \frac{80}{r^2}\cdot r^2$$

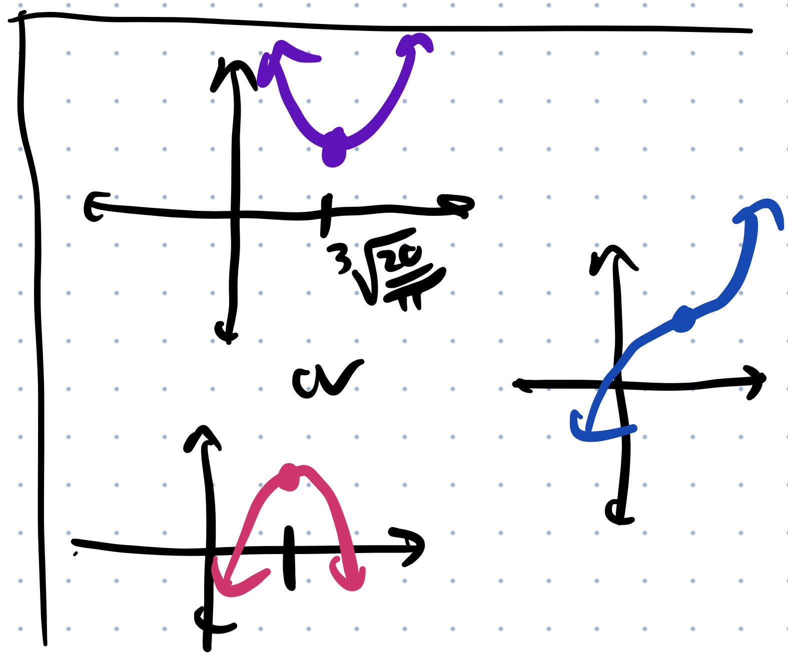
$$\Rightarrow 4\pi r^3 = 80$$

$$\Rightarrow r^3 = \frac{20}{\pi}$$

$$\Rightarrow r = \sqrt[3]{\frac{20}{\pi}}$$

positive

this is a crit. pt.



$$S(r) = 2\pi r^2 + \frac{80}{r}$$

$$r = \sqrt[3]{\frac{20}{\pi}}$$

$$S'(r) = 4\pi r + 80\cdot(-1\cdot r^{-2})$$
$$= 4\pi r - \frac{80}{r^2}$$

$80r^{-2}$
$-2\cdot 80 r^{-3}$

$0 < r < \infty$

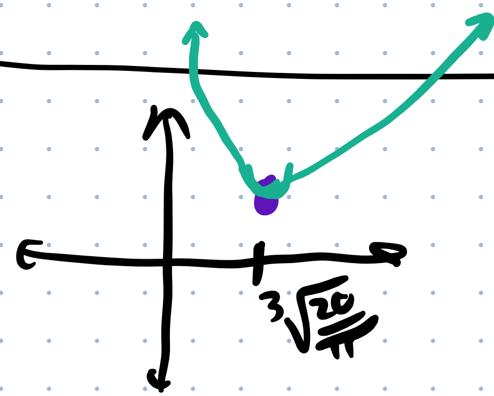
$$S''(r) = 4\pi + \frac{160}{r^3}$$

$$S''\left(\sqrt[3]{\frac{20}{\pi}}\right) = 4\pi + \frac{160}{20/\pi}$$ which is definitely positive

so this critical point is a local minimum

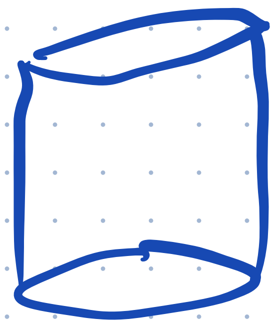
Why does this imply its actually a global min as well?

Because this is the only critical point so it can never go lower at any other point



Global minimum at $r = \sqrt[3]{\frac{20}{\pi}} \approx 1.85$

$$h = \frac{40}{\pi r^2} = \frac{40}{\pi\left(\sqrt[3]{\frac{20}{\pi}}\right)^2} \approx 3.7$$



Summary of the Steps:

(1) Identify the constraint and the quantity you are optimizing.

(2) Solve the constraint for one variable in terms of the other

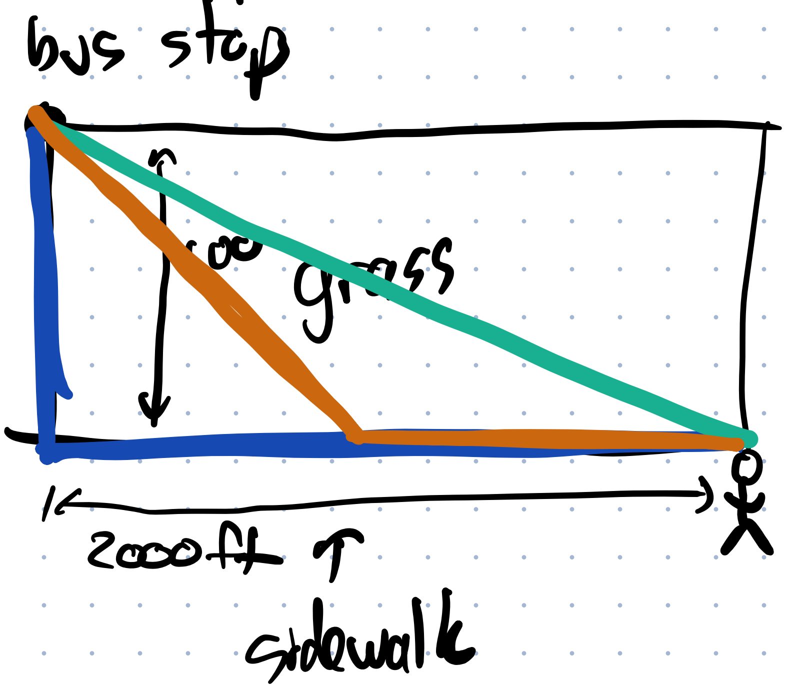
(3) Plug in to get a function in only one variable

(4) Optimize using the methods from 4.1 and 4.2

Ex (setup only)

Alama wants to get to the bus stop as quickly as possible. The bus stop is across a grassy park 2000 ft west and 600 ft north of her starting point. Speed walking in grass: 4 ft/sec
Speed walking on sidewalk: 6 ft/sec.

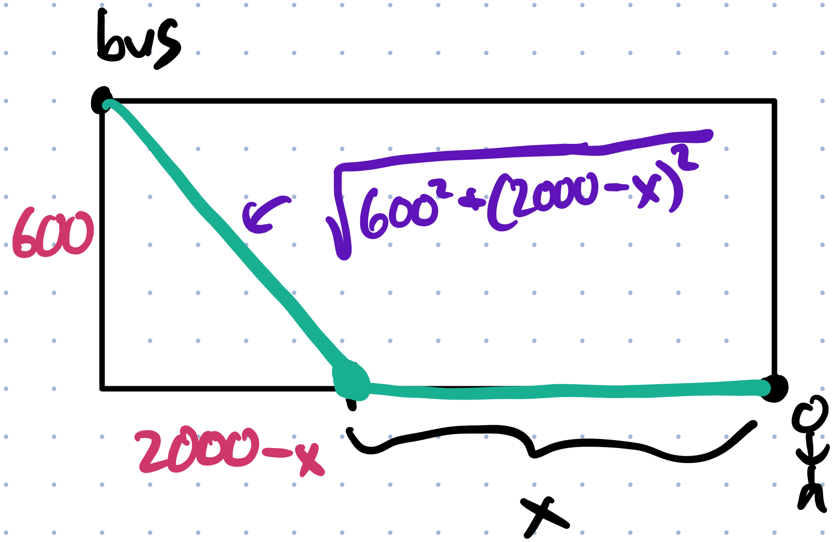
What path to the bus stop is fastest?



= longest, but most time on sidewalk

= shortest distance, but all on the slow grass

= an infinite # of poss. in the middle



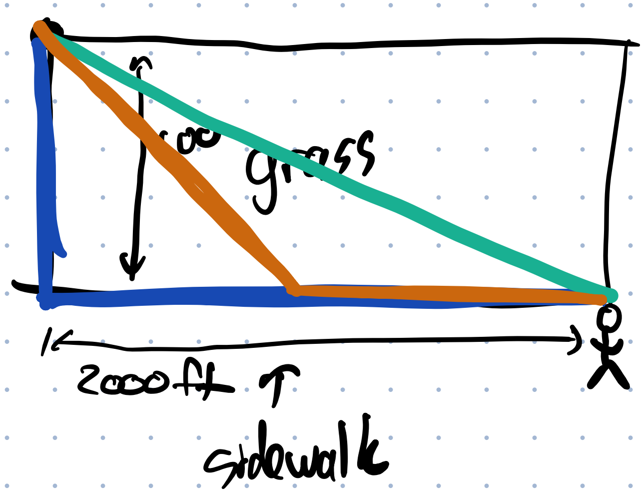
$$0 \le x \le 2000$$

Time taken: $\underbrace{\dfrac{x}{6} + \dfrac{\sqrt{600^2+(2000-x^2)}}{4}}_{\text{Minimize this}}$

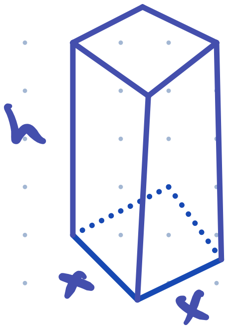
(1) critical points

(2) endpoints

answer $\boxed{\begin{aligned} x &= 2000 - 240\cdot\sqrt{5} \\ &\approx 1463 \text{ ft} \end{aligned}}$



Ex: What is the maximum volume of a closed box (all 6 sides) with a square base and exactly 24 in^2 surface area?



Volume: $x^2 \cdot h$

Surface area: $2x^2 + 4xh = 24$

constraint

$$\Rightarrow h = \frac{24 - 2x^2}{4x}$$

volume: $x^2 \cdot \left(\frac{24 - 2x^2}{4x}\right)$

maximize