Math 1450 – Calculus 1

Wed, Oct. 29

Announcements:

* HW 9 due Thursday, 3.7 – 3.9
* Quiz 7 Thursday, covers sugg. HW from last Fri, Mon, and today

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 4.1: Using First and Second Derivatives

Section 4.1 – Using first and second derivatives

4.1 + 4.2 = using f' and f'' to learn about f

Summary of known facts

- * If $f' > 0$, then f is increasing.
- * If $f' < 0$, then f is decreasing.

bending upward

- * If $f'' > 0$, then f is concave up
- * If $f'' < 0$, then f is concave down

Ex: Analyze $f(x) = x^3 - 9x^2 - 48x + 52$.

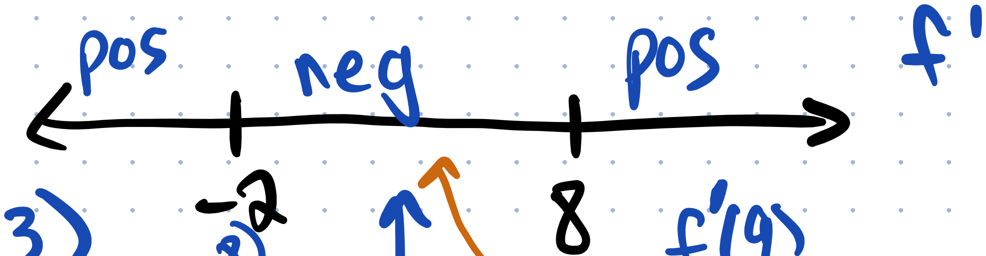
General shape of a cubic polynomial. 

$f'(x) = 3x^2 - 18x - 48$.

$= 3(x^2 - 6x - 16) = 3(x+2)(x-8)$ 

$f'(x) = 0$ at $x = -2$, and $x = 8$

Where is f' positive versus negative?



$f'(-3) = 3 \cdot (-3+2) \cdot (-3-8) = p \cdot n \cdot n = p$

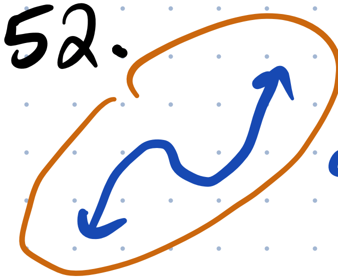
$f'(0) = 3 \cdot (0+2) \cdot (0-8) = \text{pos} \cdot \text{pos} \cdot \text{neg} = \text{ne}$

$f'(9) = 3 \cdot (9+2) \cdot (9-8) = p \cdot p \cdot p = p$

In each of the three regions of the #, f' is all positive or all negative — it can't switch in the middle of a region

Ex: Analyze $f(x) = x^3 - 9x^2 - 48x + 52$.

General shape of a cubic polynomial.



or

$f'(x) = 3x^2 - 18x - 48$

$f''(x) = 6x - 18 = 6\cdot(x-3)$

f'' $f''(x) = 0$ when $x = 3$

neg | 3 | pos

$f''(0)$
$= 6\cdot(0-3)$
$=$ neg

$f''(4)$
$= 6\cdot(4-3)$
$=$ pos

f'

pos | -2 | neg | 8 | pos

Ex: Analyze $f(x) = x^3 - 9x^2 - 48x + 52$.

General shape of a cubic polynomial.

f'

pos	-2	neg	8	pos

f''

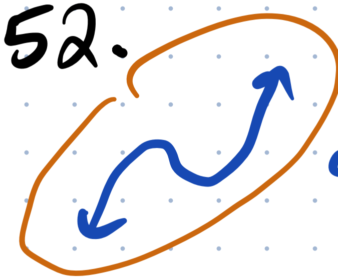
neg	3	pos

Combine these two number lines:

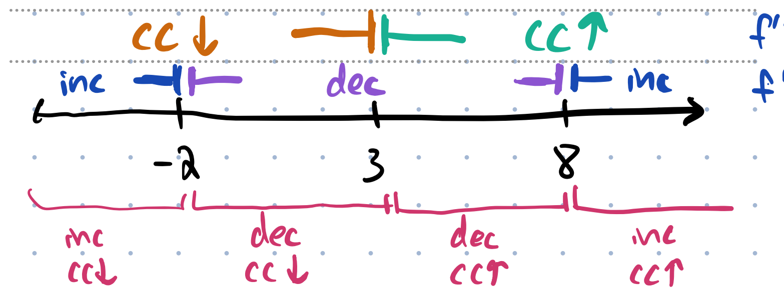
	$(-\infty, -2)$	-2	$(-2, 3)$	3	$(3, 8)$	8	$(8, \infty)$
f''	CC↓						CC↑
f'	inc		dec				inc
	inc CC↓		dec CC↓		dec CC↑		inc CC↑

Also note that $f(0) = 52$

Ex: Analyze $f(x) = x^3 - 9x^2 - 48x + 52$.

General shape of a cubic polynomial.  or 

Combine these two number lines:



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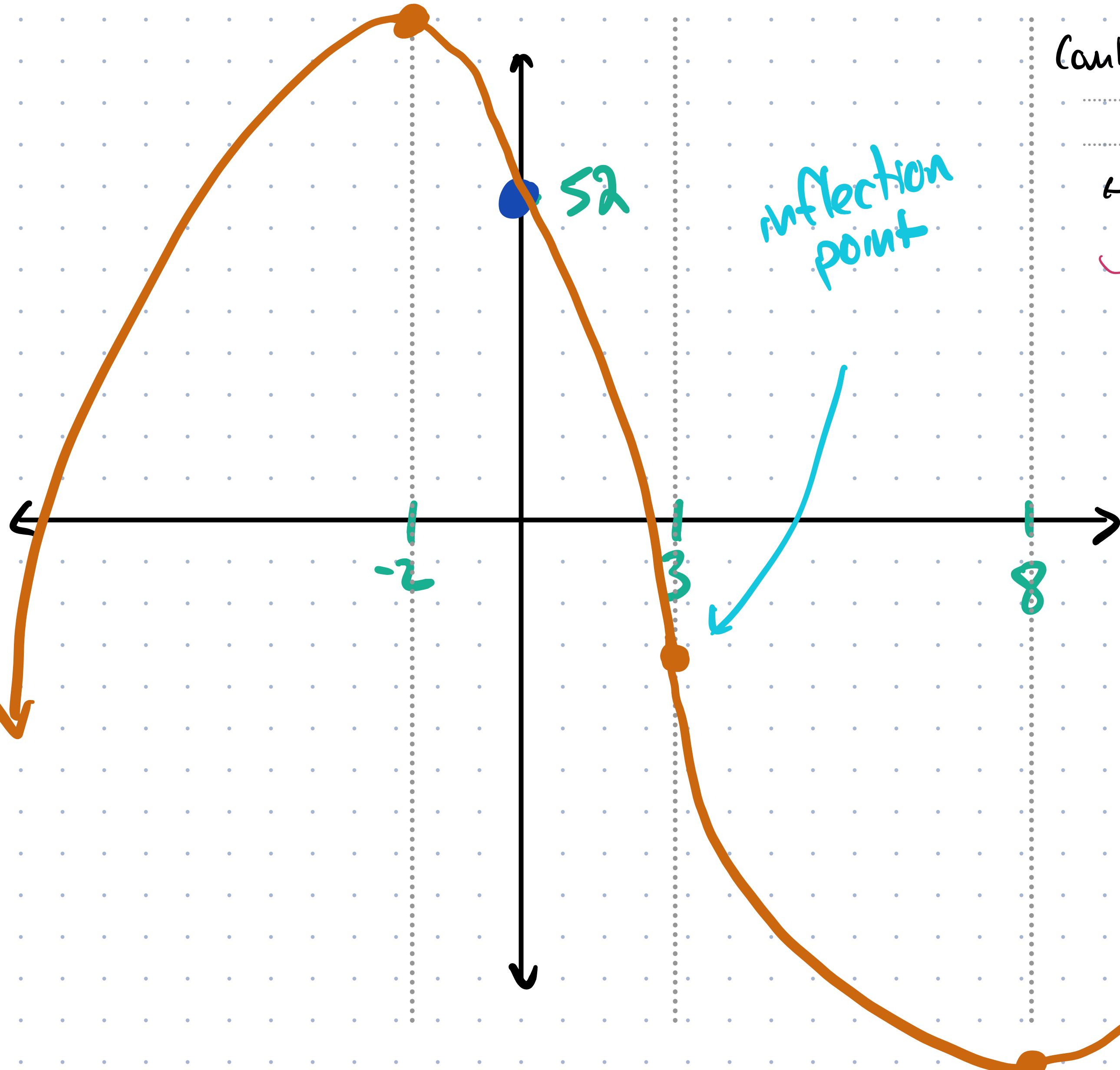
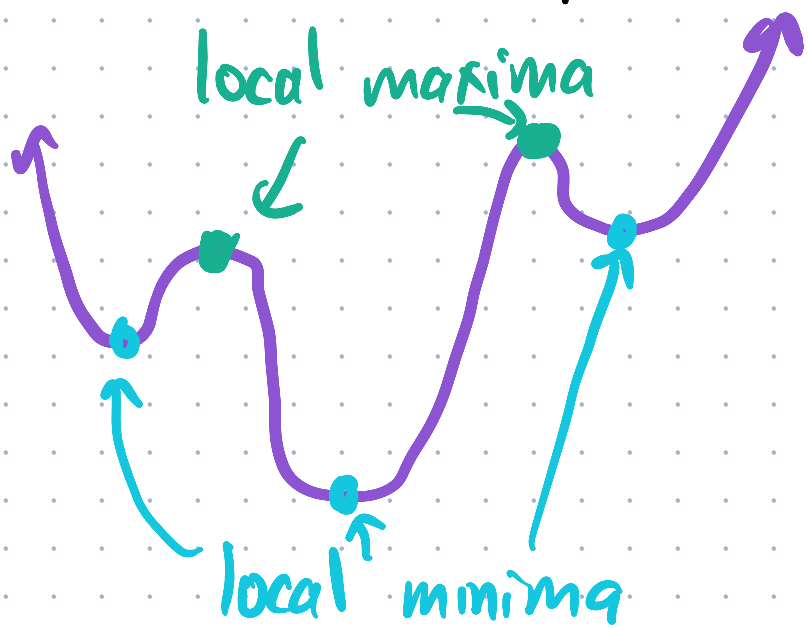


Local Maxima and Minima

(local vs. global)

* A local minimum is a point of a function whose value is smaller than all of the points near it. (a valley)

* A local maximum is a point of a function whose value is larger than all of the points near it. (a peak)



"maxima" is the plural of maximum

same for minima

Finding local minima and maxima:

Idea: When there's a local max or min, the derivative must be 0 or undefined.

Definition:

A critical point is an x-value $x = p$ in the domain of $f(x)$ where either:

* $f'(p) = 0$

OR

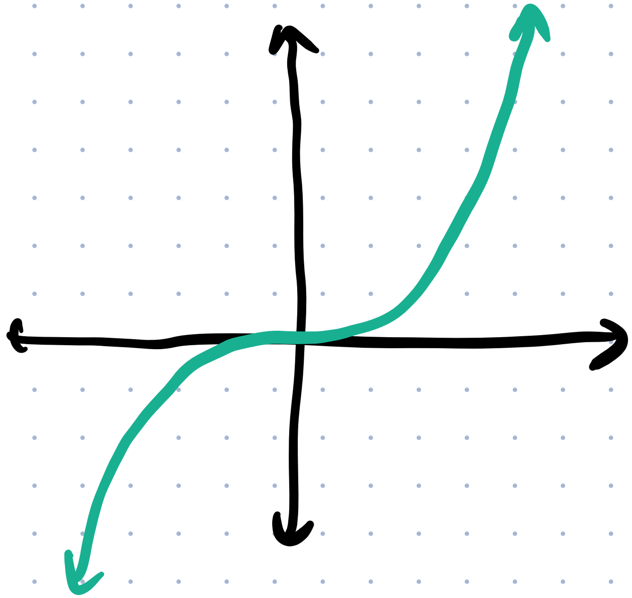
* $f'(p)$ is undefined

Theorem: All local extrema occur at critical points.

maxima or minima

Warning: not all critical points are local extrema!!

$f(x) = x^3$



$f'(x) = 3x^2$

$x=0$ is a crit. pt.

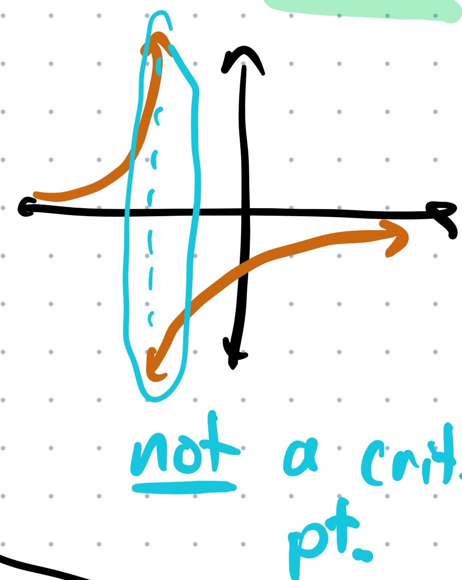
but $x=0$ is neither a local max nor a local min

Algorithm for finding local extrema of $f(x)$.

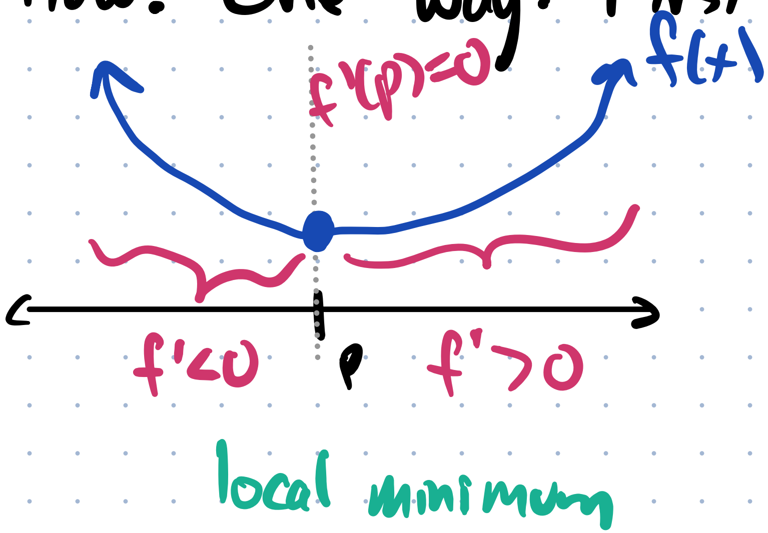
(1) Find the critical points (candidates)

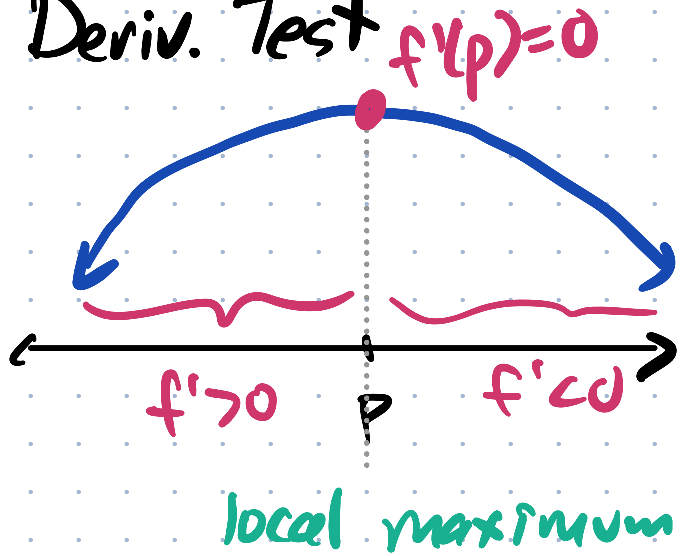
- compute $f'(x)$
- solve for where $f'(x)=0$
- also include where f' is undefined but f is defined

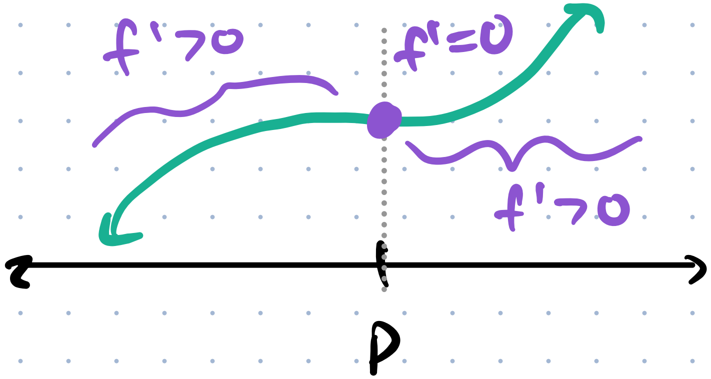
(2) Check each critical point to see if it's a local min, local max, or neither.



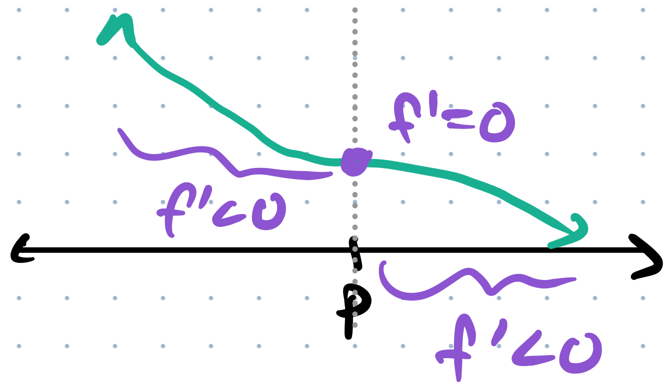
How? One way: First Deriv. Test







neither local max nor a local min



neither

How will you determine whether f' is $+/-$ on either side?

The whole number line thing

Ex: $f(x) = \dfrac{1}{x(x-1)}$