Math 1450 - Calculus 1

Mon, Oct. 27

Announcements:

* HW 9 due Thursday, 3.7 - 3.9
* Quiz 7 Thursday, covers sugg. HW from last Fri, today, and Wed

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

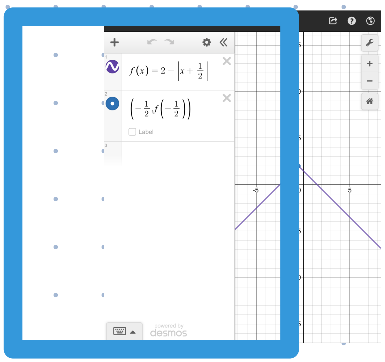
→ 3.9: Linear Approximations
→ 3.10: Theorems about Differentiable Functions
→ 4.1: Using First and Second Derivatives

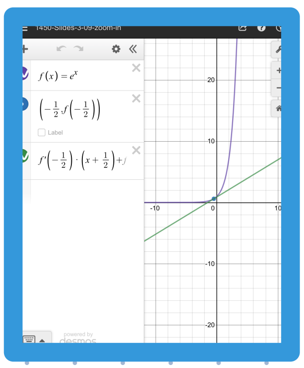
Not covering 3.8.

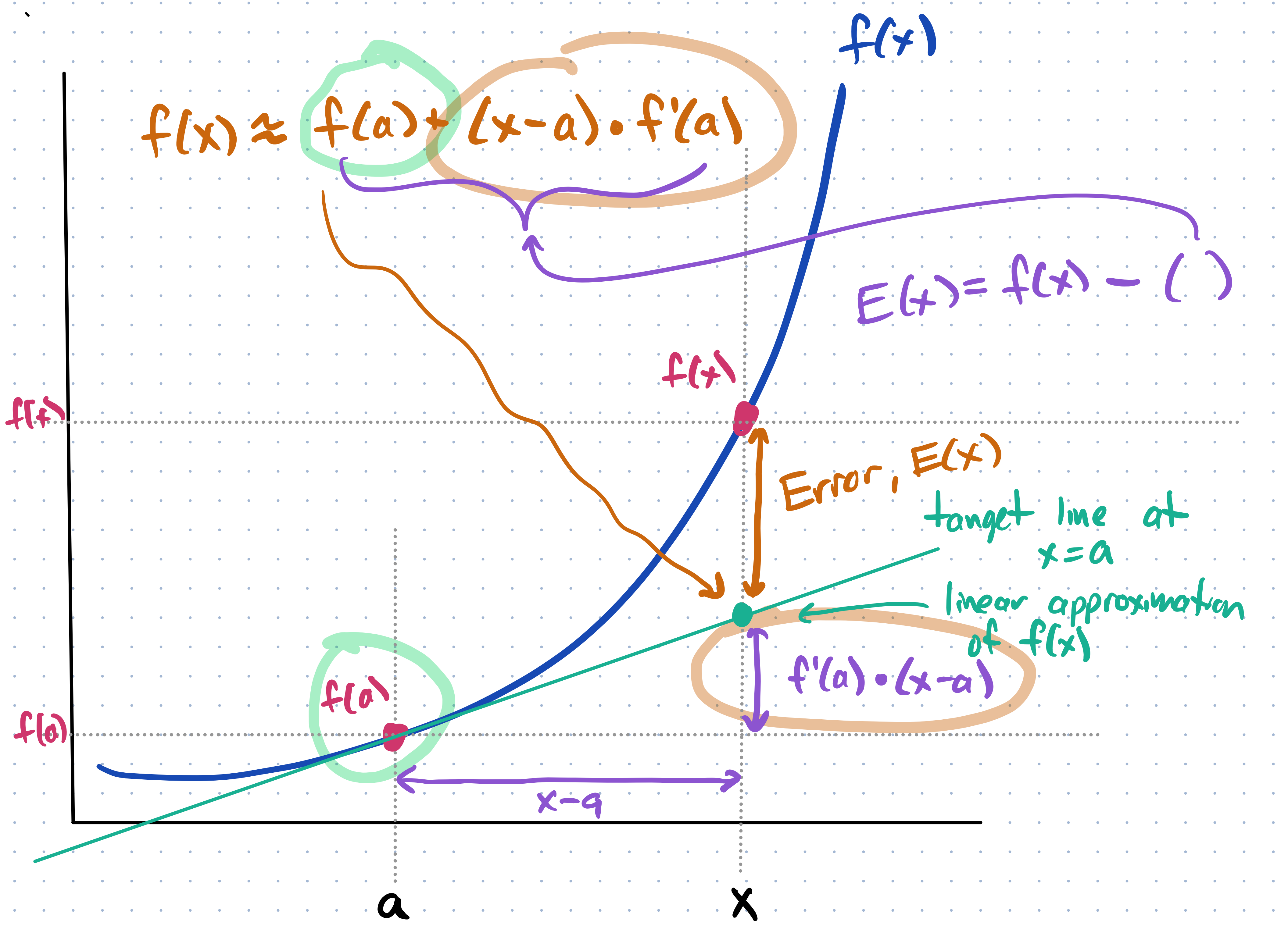
3.9: Linear Approximations and the Derivative

If you zoom in very close on any function at an x-value where it is <u>differentiable</u>, then it looks like a straight line.

<u>Take away:</u> The tangent line at a point is a pretty good approximation of the function as long as you don't look too far away from the point.







f(x)
f(x) ≈ f(a) + (x−a) · f′(a)
E(x) = f(x) − ( )
f(x)
f(x)
Error, E(x)
tangent line at x=a
linear approximation of f(x)
f′(a) · (x−a)
f(a)
f(a)
x−a
a
x

Reminder:

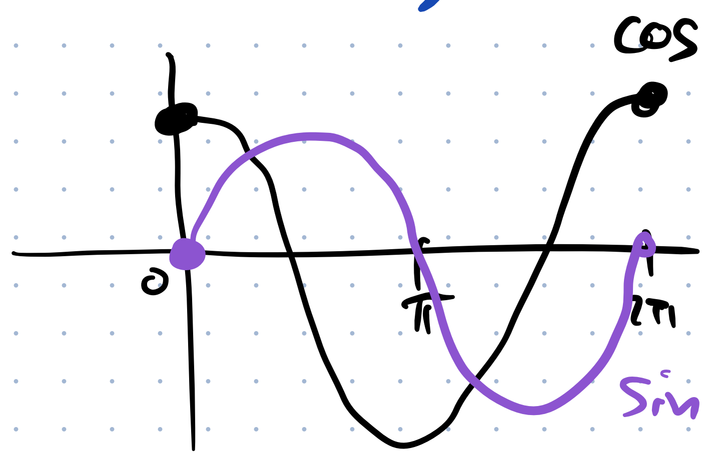
$$\underline{f(a) + (x-a)\cdot f'(a)}$$

is just another way of writing the T.L at $x=a$

Ex: What is the linear approximation of

$g(t) = \sin(t)$ ~~at~~ near $t = 0$.

(Same question as: what is the T.L. at $t=0$)



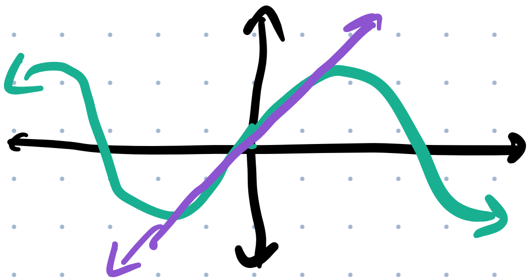
$g'(t) = \cos(t)$

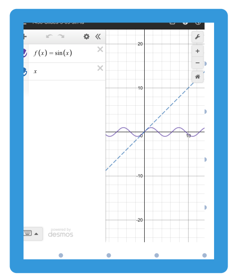
$g'(0) = \cos(0) = 1$ slope

$g(0) = \sin(0) = 0$ point

slope of 1, goes through (0,0)

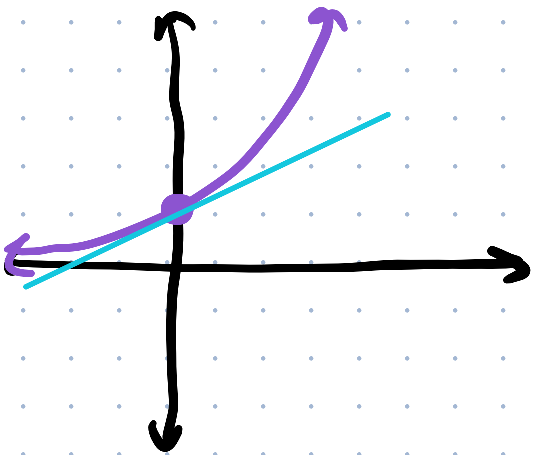
linear approximation: $\sin(t) \approx t$ near $t=0$





Ex: What is the linear approximation of

$f(x) = e^{k\cdot x}$, near $x=0$
at



$$f'(x) = k \cdot e^{kx}$$

$$f'(0) = k \cdot e^{k\cdot 0} = k \cdot e^{0} = k$$

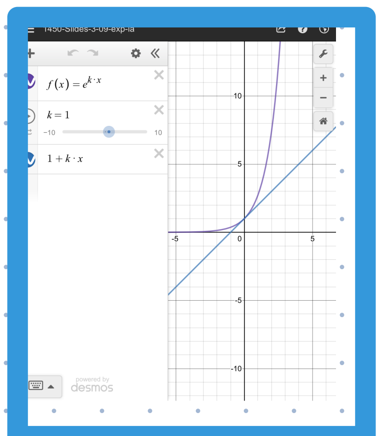
$$f(0) = e^{k\cdot 0} = e^{0} = 1$$

Linear approx: slope k

point $(0,1)$

$$e^{k\cdot x} \approx k\cdot x + 1$$

$$m\cdot x + b$$



How good are these approximations?

$k=3$

x	$\sin(x) \approx x$
0	0
0.01	0.0099998
0.05	0.04997
0.1	0.0998
0.5	0.479
1	0.84

very good

x	e^{3x}	$3x+1$
0	1	1
0.01	1.0304...	1.03
0.05	1.1618...	1.15
0.1	1.349...	1.3
0.5	4.48...	2.5
1	20.08...	4

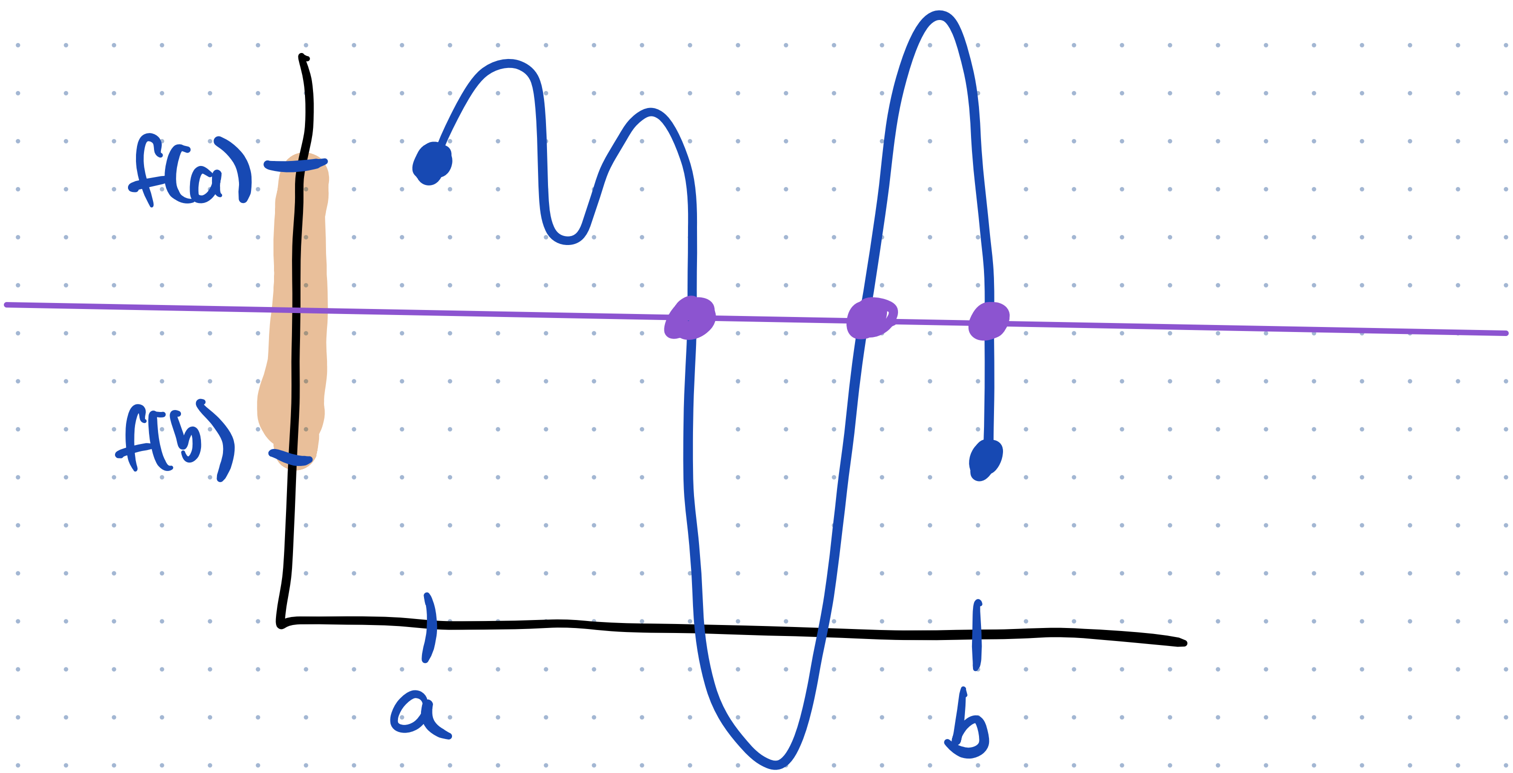
pretty good

very bad

3.10 - Theorem~~s~~ about Differentiable Functions

From Section 1.7: Intermediate Value Theorem

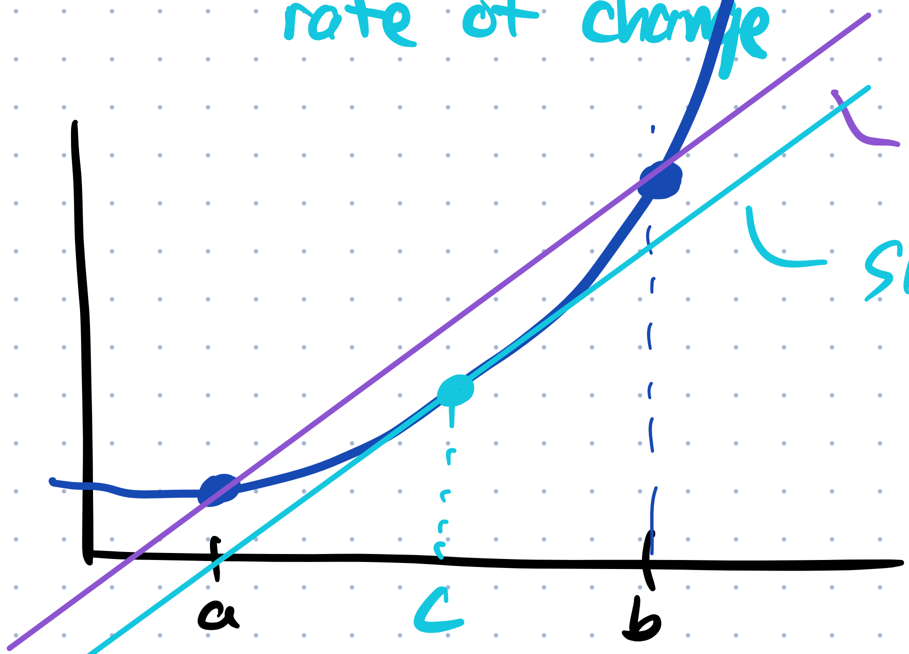
If f is continuous on the interval $[a,b]$ then every value between $f(a)$ and $f(b)$ is reached somewhere in the middle.



New: Mean Value Theorem

If f is <u>differentiable</u> on an interval $[a,b]$, then there exists some number c between a and b such that

$$\underbrace{f'(c)}_{\text{instantaneous rate of change}} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{average rate of change between } a \text{ and } b}.$$



slope =

slope =

Ex: If you drive 180 miles in 3 hours, then at some moment you must have been traveling <u>exactly</u> 60 mph.

$$\frac{180}{3} = 60$$ avg rate over 3 hours

Section 4.1 – Using first and second derivatives

4.1 + 4.2 = using f' and f'' to learn about f

Summary of known facts

* If $f' > 0$, then f is increasing.
* If $f' < 0$, then f is decreasing.

bending upward

* If $f'' > 0$, then f is concave up
* If $f'' < 0$, then f is concave down

Ex: Analyze $f(x)= x^3-9x^2-48x+52$.

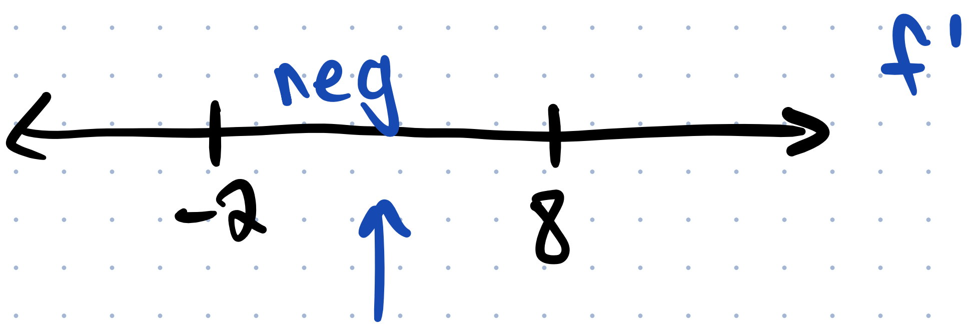
General shape of a cubic polynomial.

$$f'(x)=3x^2-18x-48.$$

$$=3(x^2-6x-16)=3(x+2)(x-8)$$

$f'(x)=0$ at $x=-2$, and $x=8$

Where is f' positive versus negative?



$f'(0)=$ neg