Math 1450 - Calculus 1

Fri. Oct. 24

Announcements:

* HW 9 due next Thursday, 3.7 + 3.9
* Quiz 7 next Thursday

Today:

→ 3.7: Implicit Functions
→ 3.9: Linear Approximations

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Implicit functions are defined by equations involving x and y.

(something with an "=" sign

Ex: $x^2+y^2=4$

not possible to write as $y=[\quad]$

Ex: $y^3-x\cdot y=-6$

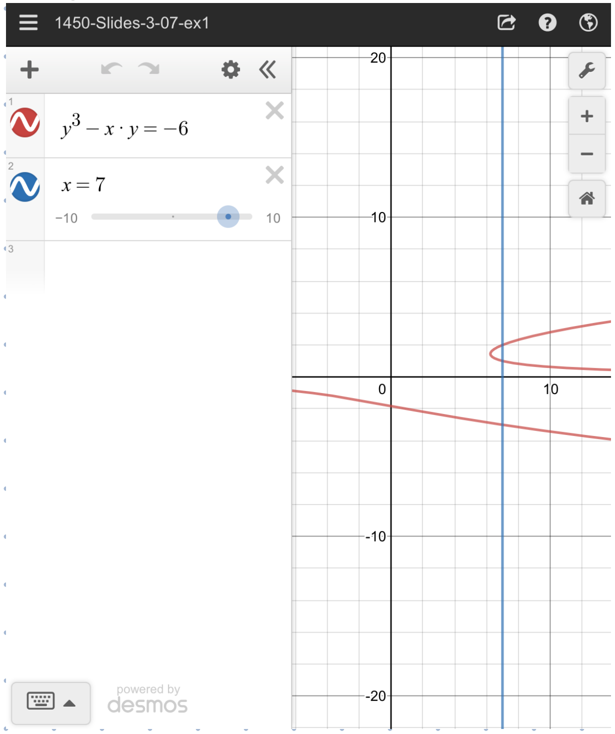
This means "the infinite set of (x,y) points for which the equation $y^3-xy=-6$ is true".

Is $(2,4)$ on this graph?

$4^3-2\cdot 4 \stackrel{?}{=} -6$

$56 \stackrel{?}{=} -6$ No, $(2,4)$ is not on this graph.

$$y^3 - xy = -6$$



$(7,-3)$ → $(-3)^3 - 7\cdot(-3)$

$-27+21=-6$ ✓

$(7,1)$ → $1^3 - 7\cdot(1)$

$1-7=-6$ ✓

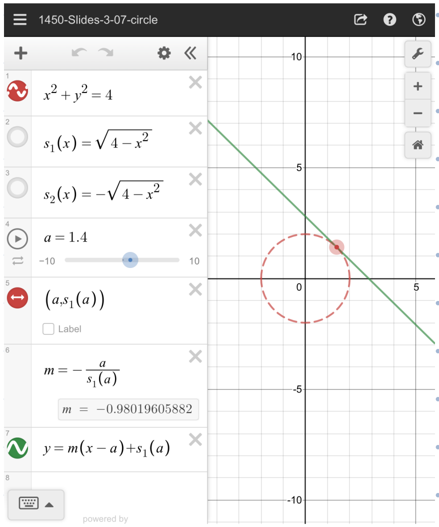
$(7,2)$ → $2^3 - 7\cdot 2$

$=8-14=-6$ ✓

How do we find slopes of tangent lines of implicit functions?

Chain Rule

Ex: $x^2+y^2=4$ circle centered at $(0,0)$ with radius 2



Take the derivative of both sides:

$$\underbrace{\frac{d}{dx}\left(x^2+y^2\right)} = \cancel{\frac{d}{dx}(4)}$$

0

$y(x)$

$$\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^2\right)=0$$

$$2x+\underbrace{\frac{d}{dx}\left(y^2\right)}=0$$

$2\cdot y\cdot y'$

$$\boxed{\frac{d}{dx}\left(\left(f(x)\right)^2\right) = 2\cdot f(x)\cdot f'(x)}$$

$$2x+2\cdot y\cdot y'=0$$

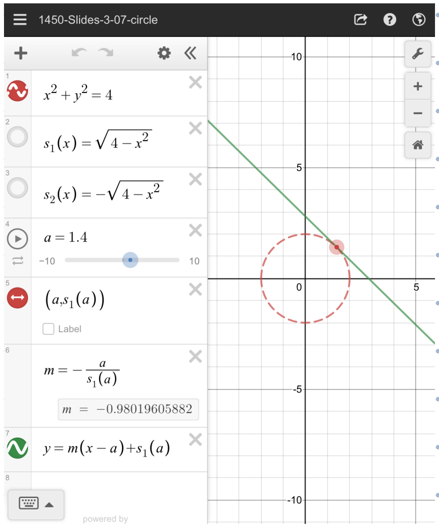
Solve for y':

$$2\cdot y\cdot y' = -2x$$

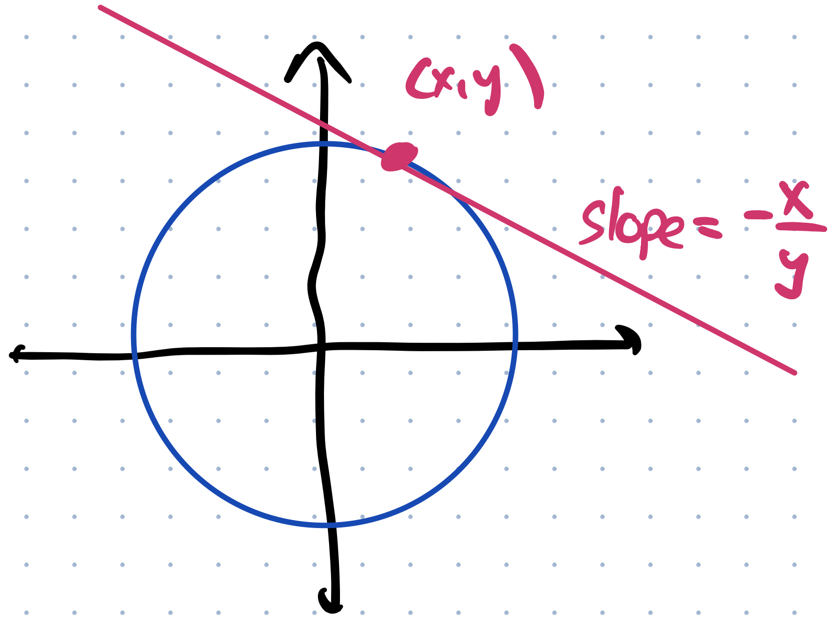
$$y' = -\frac{2x}{2y} \longrightarrow y' = -\frac{x}{y}$$

Ex: $x^2+y^2=4$

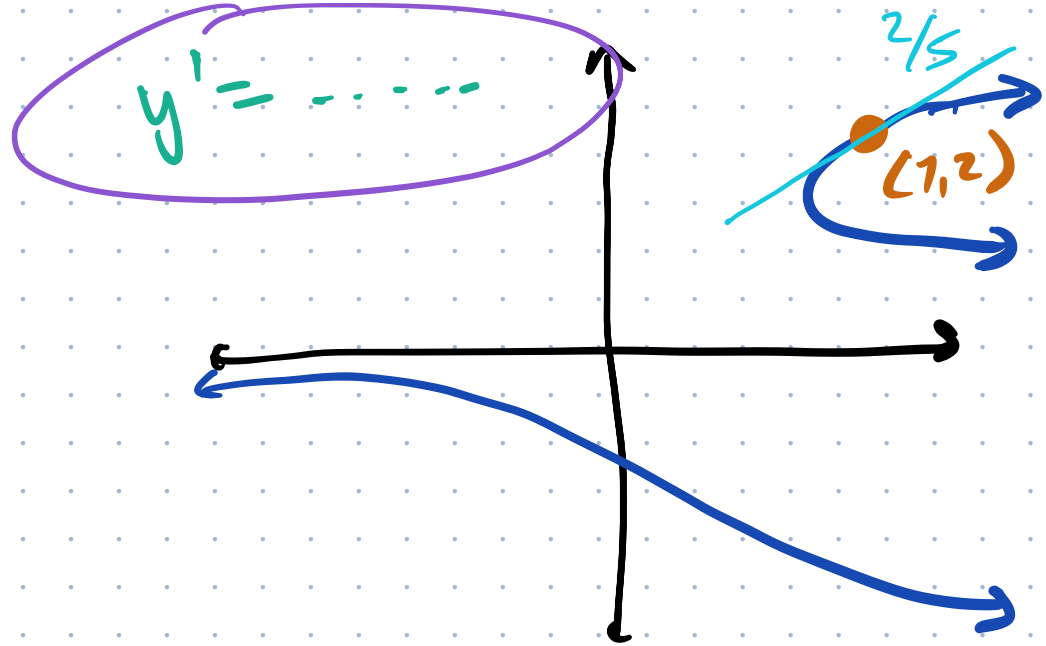
circle centered at (0,0) with radius 2



$$y' = -\frac{x}{y}$$



Ex: $y^3 - xy = -6$



$$\frac{d}{dx}(y^3 - xy) = \cancel{\frac{d}{dx}(-6)}\ 0$$

$$\underbrace{\frac{d}{dx}(y^3)} - \frac{d}{dx}(\overset{f}{x}\overset{g}{y}) = 0$$

$$3y^2 \cdot y' - (1 \cdot y + x \cdot y') = 0$$

$$\frac{d}{dx}(f \cdot g) = f' \cdot g + f \cdot g'$$

Solve for y': get all terms with y' on one side, all terms without on the other side

$$3y^2y' - y - xy' = 0$$

$$\Rightarrow 3y^2y' - xy' = y$$

$$\Rightarrow y' \cdot (3y^2 - x) = y$$

$$\boxed{y' = \frac{y}{3y^2 - x}}$$

(7,2)

$$\frac{2}{3 \cdot 2^2 - 7} = \frac{2}{5}$$

$$y^3 \to \underbrace{\left[(f(x))\right]^3}$$

$$3 \cdot (f(x))^2 \cdot \frac{d}{dx}$$

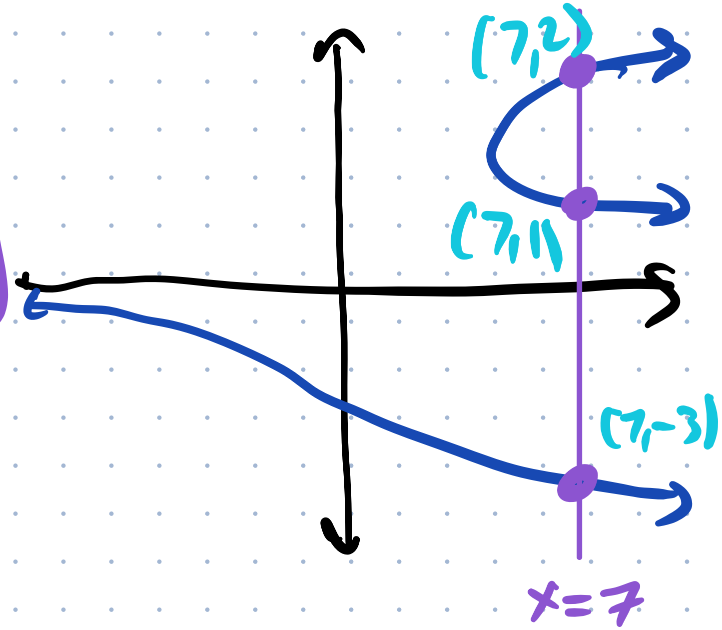
$$3 \cdot (f(x))^2 \cdot f'(x)$$

$$3 \cdot y^2 \cdot y'$$

On the implicit function $y^3 - xy = -6$, what are the tangent lines at $x = 7$?

$$y' = \frac{y}{3y^2 - x}$$

What points are on the graph at $x=7$? (what are the y-values?)



Plug in $x=7$: $y^3 - 7y = -6$

$$\Rightarrow y^3 - 7y + 6 = 0$$

$$\Rightarrow (y-1)\cdot(y-2)\cdot(y+3) = 0$$

$$\Rightarrow y = 1, 2, -3$$

$(7,1), (7,2), (7,-3)$

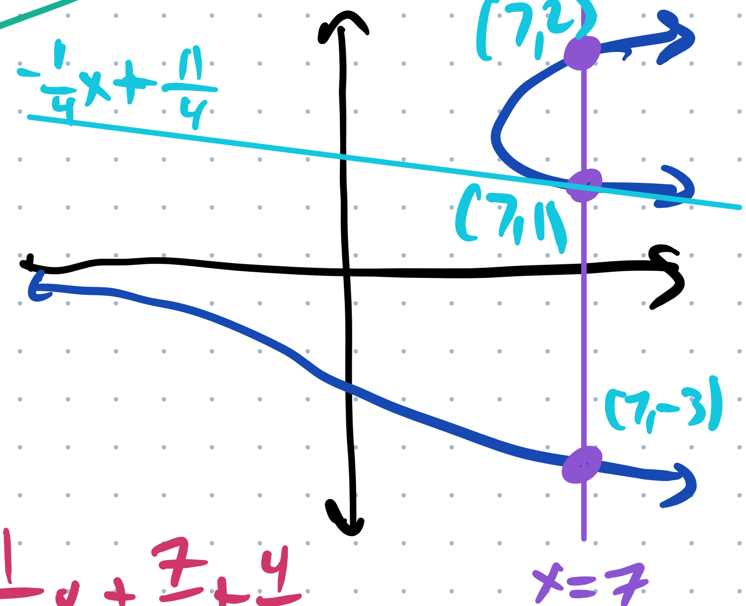
↳ check: $1^3 - (7\cdot 1) = -6$

$1 - 7 = -6$ ✓

Not done yet, but now we know the 3 points where we need to find the T.L.

On the implicit function $y^3-xy=-6$, what are the tangent lines at $x=7$?

$$y' = \frac{y}{3y^2-x}$$



(7,1), (7,2), (7,−3)

slope at $(\overset{x}{7},\overset{y}{1})$ is:

$$y' = \frac{1}{3\cdot(1)^2-7} = -\frac{1}{4}$$

point: (7,1)

slope: $-\frac{1}{4}$

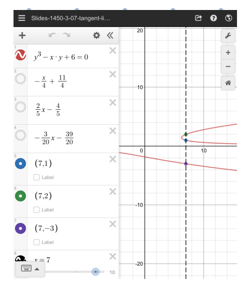
$$y=-\frac{1}{4}(x-7)+1 = -\frac{1}{4}x+\frac{7}{4}+\frac{4}{4}$$

$$\boxed{= -\frac{1}{4}x+\frac{11}{4}}$$

(7,2) slope: $\frac{2}{5}$ $\boxed{y=\frac{2}{5}x-\frac{4}{5}}$

(7,−3)

↳ slope $-\frac{3}{20}$ $\boxed{y=-\frac{3}{20}x-\frac{39}{20}}$



$(\underset{a}{7}, \underset{b}{-3})$ slope $-\frac{3}{20}$

$y = m\cdot(x-a)+b$ $\qquad$ $y-b = m\cdot(x-a)$

$$y = -\frac{3}{20}\cdot(x-7)+(-3)$$

$$= -\frac{3}{20}x + \left(\frac{3}{20}\right)\cdot 7 - \frac{60}{20}$$

$21-60=-39$

$$= -\frac{3}{20}x + \frac{21}{20} - \frac{60}{20}$$

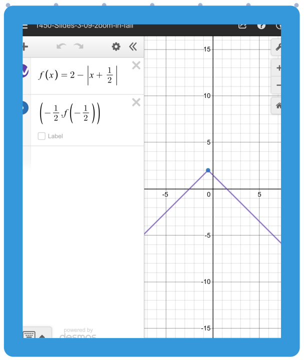
$$= -\frac{3}{20}x - \frac{39}{20}$$

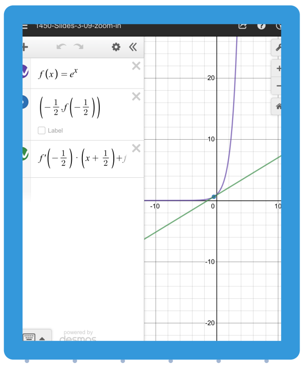
Not covering 3.8.

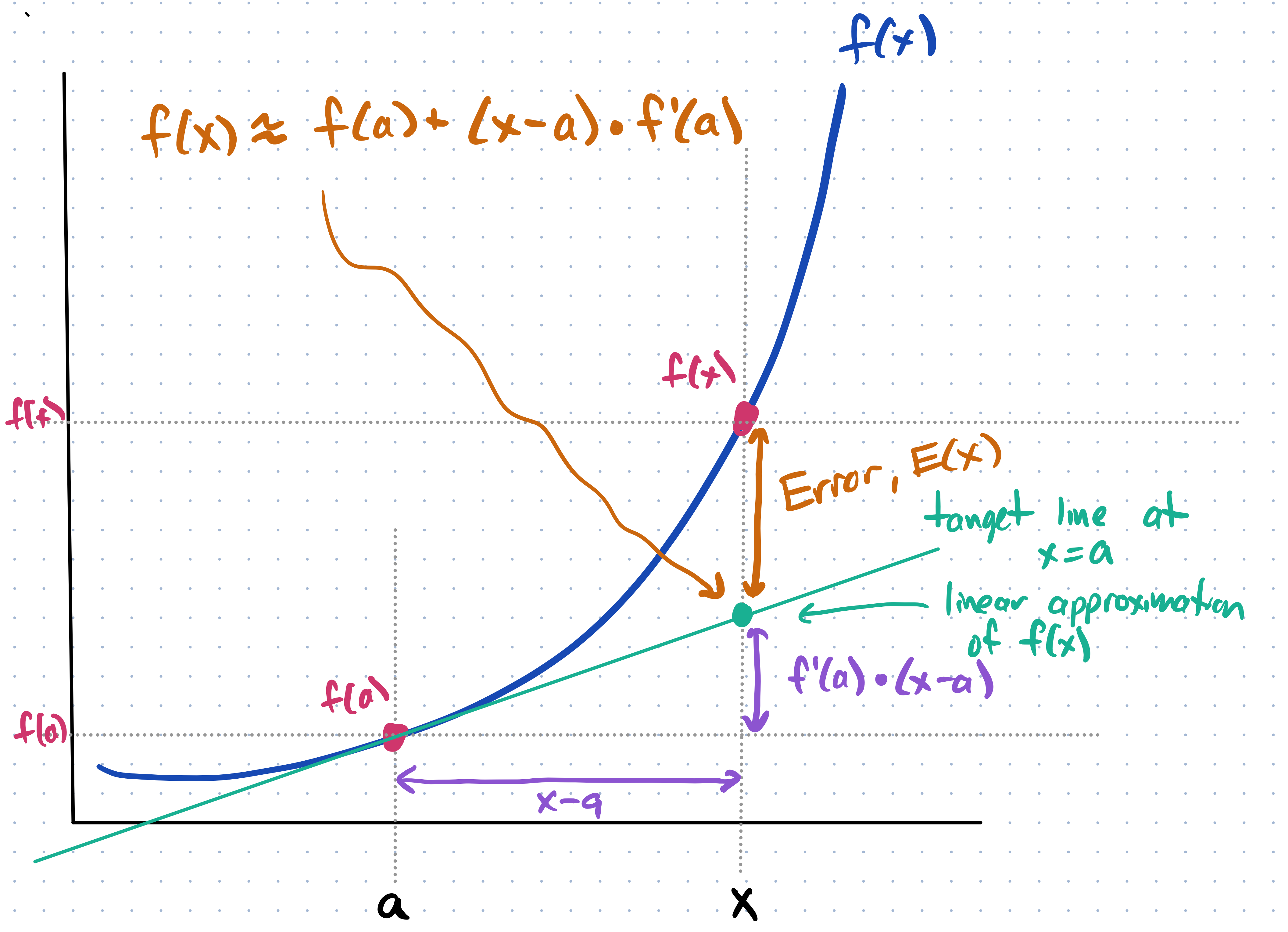
3.9: Linear Approximations and the Derivative

If you zoom in very close on any function at an x-value where it is <u>differentiable</u>, then it looks like a straight line.

<u>Take away:</u> The tangent line at a point is a pretty good approximation of the function as long as you don't look too far away from the point.







f(x)
f(x) ≈ f(a) + (x−a) · f'(a)
f(x)
f(x)
Error, E(x)
tangent line at
x=a
linear approximation
of f(x)
f'(a) · (x−a)
f(a)
f(a)
x−a
a
x