Math 1450 - Calculus 1

Wed, Oct. 22

Announcements:

* HW 8 due on Thursday → covers 3.6 only
* Q6 on Thursday, covers 3.5 + 3.6

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 3.6: Chain Rule and Inverse Functions

→ 3.7: Implicit Functions

Summary

* $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

* $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

arctan(x)

$\tan^{-1}(x)$

two names for the same thing

* $\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$ $\sin^{-1}(x)$

* $\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$ $\cos^{-1}(x)$

General Rule for finding the derivative of an inverse function.

Suppose you know $f(x)$, $f'(x)$, and $f^{-1}(x)$.
What is $(f^{-1})'(x)$?

$f(f^{-1}(x)) = x$ (always true because this is the def. of f^{-1})

Derivative of both sides:

$$\underbrace{f'(f^{-1}(x)) \cdot (f^{-1})'(x)}_{f'(f^{-1}(x))} = \frac{1}{f'(f^{-1}(x))}$$

$$\Rightarrow (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Assume f^{-1} exists.

Ex: Suppose $\boxed{f(2) = 4, \quad f'(2) = 7.}$

What is $(f^{-1})'(4)$?

$$2 \xrightarrow{\ f\ } 4 \qquad 2 \xleftarrow{\ f^{-1}\ } 4$$

Formula: $$(f^{-1})'(4) = \frac{1}{f'(\underbrace{f^{-1}(4)}_{2})}$$

$$= \frac{1}{f'(2)} = \boxed{\frac{1}{7}}$$

Logarithmic Differentiation

Let $f(x) = x^x$.

$f(1.5) = 1.5^{1.5}$

We have no way of finding f' yet.

Trick: $f(x) = x^x = e^{\ln(x^x)} = e^{x \cdot \ln(x)}$

$f'(x)$: $e^{x\cdot\ln(x)} \cdot \frac{d}{dx}(\overset{g}{x} \cdot \overbrace{\ln(x)}^{h})$ [chain rule]

$= \underbrace{e^{x\cdot\ln(x)}}_{} \cdot \left(\overset{g \cdot h'}{x \cdot \frac{1}{x}} \overset{+}{+} \overset{g' \cdot h}{1 \cdot \ln(x)}\right)$

$= x^x \cdot (1 + \ln(x))$

Ex: $\dfrac{d}{dx}\left((x^2+1)^{(\sqrt{x})}\right)$

$$= \frac{d}{dx}\left(e^{\ln\left((x^2+1)^{(\sqrt{x})}\right)}\right) = \frac{d}{dx}\left(e^{\sqrt{x}\cdot\ln(x^2+1)}\right)$$

$$= e^{\sqrt{x}\cdot\ln(x^2+1)} \cdot \frac{d}{dx}\left(\overset{f}{\sqrt{x}} \cdot \overset{g}{\ln(x^2+1)}\right)$$

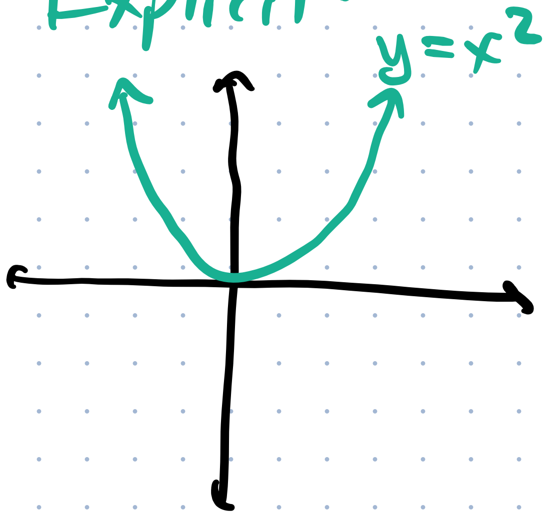
$$= e^{\sqrt{x}\cdot\ln(x^2+1)} \cdot \left(\overset{f'}{\frac{1}{2}x^{-1/2}} \cdot \overset{g}{\ln(x^2+1)} + \sqrt{x} \cdot \frac{1}{x^2+1} \cdot 2x\right)$$

$$= (x^2+1)^{\sqrt{x}} \cdot \left(\frac{\ln(x^2+1)}{2\sqrt{x}} + \frac{2x\sqrt{x}}{x^2+1}\right)$$

Section 3.7 - Implicit Functions

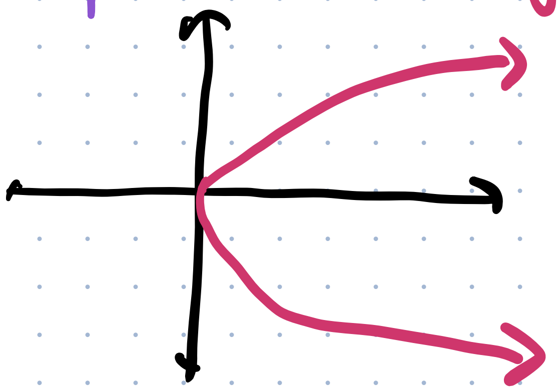
So far we've mostly dealt with explicit functions defined directly as $y = [\text{stuff with } x]$.

Explicit:



Implicit:

$x = y^2$



Implicit functions are defined by equations involving x and y.

(something with an "=" sign

Ex: $x^2+y^2=4$

not possible to write as $y=[\quad]$

Ex: $y^3-x\cdot y=-6$

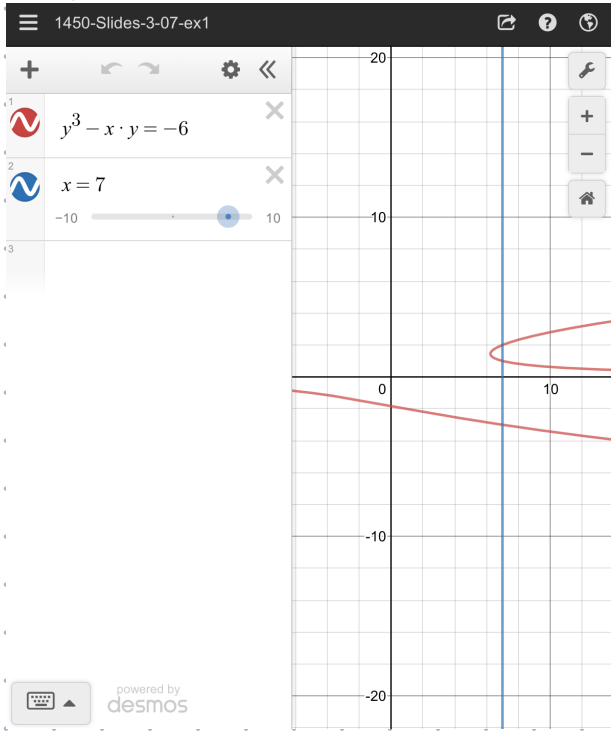
This means "the infinite set of (x,y) points for which the equation $y^3-xy=-6$ is true".

Is (2,4) on this graph?

$4^3-2\cdot 4 \stackrel{?}{=} -6$

$56 \stackrel{?}{=} -6$ No, (2,4) is not on this graph.

$$y^3 - xy = -6$$



$(7, -3) \to (-3)^3 - 7\cdot(-3)$

$-27 + 21 = -6$ ✓

$(7, 1) \to 1^3 - 7\cdot(1)$

$1 - 7 = -6$ ✓

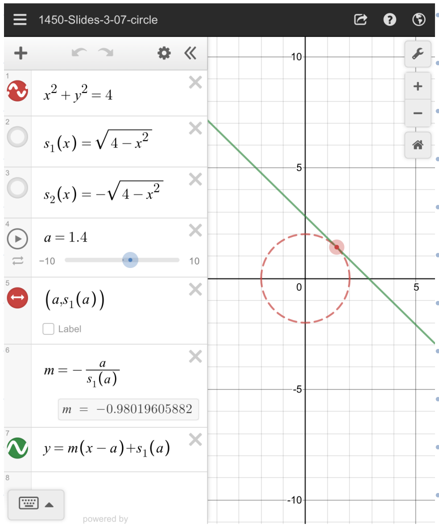
$(7, 2) \to 2^3 - 7\cdot 2$

$= 8 - 14 = -6$ ✓

How do we find slopes of tangent lines of implicit functions?

Chain Rule

Ex: $x^2+y^2=4$ circle centered at $(0,0)$ with radius 2



Take the derivative of both sides:

$$\underbrace{\frac{d}{dx}\left(x^2+y^2\right)} = \cancel{\frac{d}{dx}(4)}$$

0

$y(x)$

$$\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)=0$$

$$2x+\underbrace{\frac{d}{dx}(y^2)}_{2\cdot y\cdot y'}=0$$

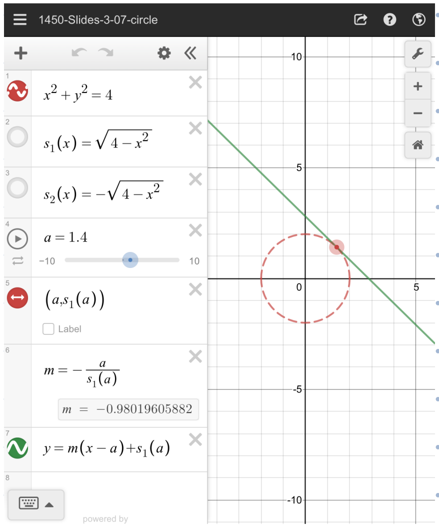
$$\frac{d}{dx}\left((f(x))^2\right) = 2\cdot f(x)\cdot f'(x)$$

$$2x+2\cdot y\cdot y'=0$$

Solve for y': $2\cdot y\cdot y' = -2x$

$$y' = -\frac{2x}{2} \longrightarrow y' = -\frac{x}{y}$$

Ex: $x^2+y^2=4$ circle centered at $(0,0)$ with radius 2



$$y' = -\frac{x}{y}$$