Math 1450 - Calculus 1

Mon, Oct. 20

Announcements:

* Midterm grades are in
  → Get in contact with me if you want to discuss!
* HW 8 due on Thursday → covers 3.6 only
* Q6 on Thursday, covers $\underbrace{3.5}_{10/10}$ + $\underbrace{3.6}_{\text{today + maybe Wed}}$

Office Hours

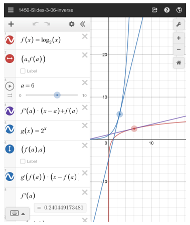
Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Today:

→ 3.6: Chain Rule and Inverse Functions

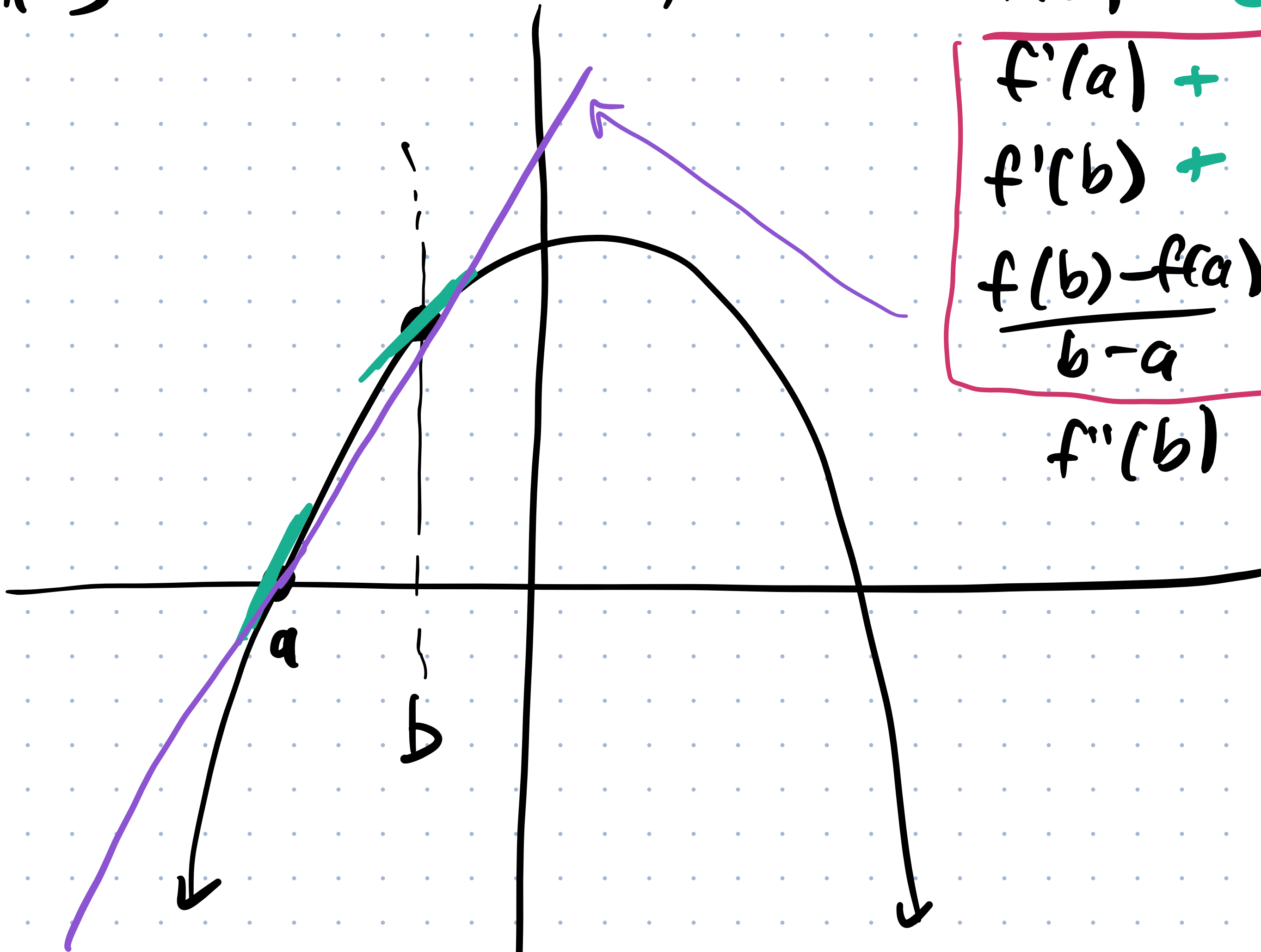


#3 smallest to largest

$f(a) = 0$

$\boxed{\begin{array}{ll} f'(a) & + \\ f'(b) & + \\ \dfrac{f(b)-f(a)}{b-a} & + \end{array}}$

$f''(b)$ −



$$f''(b) < f(a) < f'(b) < \frac{f(b)-f(a)}{b-a} < f'(a)$$

$$f(x) = \frac{15}{x+3} \qquad f'(2)$$

$$f'(x) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{15}{5+h} - 3\frac{(5+h)}{(5+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{15 - 3(5+h)}{5+h}}{h} = \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(5+h)}$$

$$= -\frac{3}{5}$$

$$\boxed{\frac{x^3+\sqrt{x}+7}{\sqrt{x}}}$$

$$\frac{x}{x^3+\sqrt{x}+7}$$

$$= \frac{x^3}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} + \frac{7}{\sqrt{x}} = x^{5/2} + 1 + 7x^{-1/2}$$

$$\rightsquigarrow \frac{5}{2}x^{3/2} - \frac{7}{2}x^{-3/2}$$

3.6: The Chain Rule and Inverse Functions

Assume you know $\frac{d}{dx}(x^2) = 2x$.
Pretend you don't know the deriv. of $\sqrt{x}$.

Define $f(x) = \sqrt{x}$. Square both sides:

$$(f(x))^2 = x$$

Take the deriv. of both sides:

$$\frac{d}{dx}\left((f(x))^2\right) = \frac{d}{dx}(x)$$

$$2 \cdot f(x) \cdot f'(x) = 1$$

plug in $f(x)$

Solve for $f'(x)$: $f'(x) = \frac{1}{2 \cdot f(x)} = \frac{1}{2 \cdot \sqrt{x}} = \frac{1}{2} x^{-1/2}$

Derivative of $\ln(x)$: Follow the same steps.

$$f(x) = \ln(x)$$

Do e to the power of each side

$$e^{f(x)} = \underbrace{e^{\ln(x)}}_{x}$$

Take deriv:

$$\frac{d}{dx}\left(e^{f(x)}\right) = \frac{d}{dx}(x)$$

$$e^{f(x)} \cdot f'(x) = 1$$

Solve for $f'(x)$:

$$f'(x) = \frac{1}{e^{f(x)}}$$

Plug in $f(x)$:

$$\boxed{f'(x) = \frac{1}{e^{\ln(x)}} = \frac{1}{x}}$$

Fact: $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

Derivatives of the inverse trig functions

<u>arctan</u> $\qquad f(x) = \arctan(x)$

Take tan of both sides:

$$\tan(f(x)) = \underbrace{\tan(\arctan(x))}_{=x}$$

Take deriv:

$$\frac{1}{\cos^2(f(x))} \cdot f'(x) = 1$$

Solve for $f'(x)$:

$$f'(x) = \cos^2(f(x))$$

Plug in $f(x)$:

$$f'(x) = \cos^2(\arctan(x))$$
$$= [\cos(\arctan(x))]^2 \rightarrow \left[\frac{1}{\sqrt{1+x^2}}\right]^2 = \boxed{\frac{1}{1+x^2}}$$

arcsin(x)

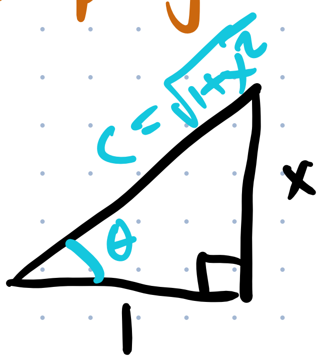
arccos(x)

$\arctan(\tan(x)) = x$

$\tan(\arctan(x)) = x$

Simplify $\cos(\underbrace{\arctan(x)}_{\theta})$

$\tan(\theta) = x$



$$\cos(\quad) = \frac{1}{\sqrt{1+x^2}}$$

$$(1)^2 + x^2 = c^2 \Rightarrow \sqrt{1+x^2}$$

$f(x) = \arcsin(x)$

$\sin(f(x)) = \underbrace{\cancel{\sin(\arcsin}(x))}_{x}$

Deriv:

$\cos(f(x)) \cdot f'(x) = 1$

Solve for f':

$$f'(x) = \frac{1}{\cos(f(x))} = \frac{1}{\cos(\underbrace{\arcsin(x)}_{\theta,\ \text{such that } \sin(\theta) = x})}$$

$a^2 + b^2 = c^2$

$x^2 + b^2 = 1^2$

$b^2 = 1 - x^2$

$b = \sqrt{1-x^2}$

$\cos(\theta) = \sqrt{1-x^2} \implies f'(x) = \frac{1}{\sqrt{1-x^2}}$

Summary

* $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

* $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

* $\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$

* $\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$