Math 1450 - Calculus 1

Friday, Oct. 10

Announcements:

* Homework 7 due Tuesday night, covers 3.4 + 3.5
* Exam 2 is next Wed, 5pm-6pm, this room
  covers up to 3.4, no 3.5

Today:

-> 3.4: The Chain Rule
-> 3.5: Trigonometric Functions

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Double chain rule from last time:

$$\frac{d}{dx}\left((e^{-x/7}+5)^{1/2}\right)$$

This time with the "cloud rule"

$$\frac{d}{dx}\left(\left(e^{-x/7}+5\right)^{1/2}\right)$$

$-\frac{x}{7} = \left(-\frac{1}{7}\right)\cdot x$

$\frac{d}{dx}\left(e^{-x/7}\right) + \frac{d}{dx}(5)$, $\frac{d}{dx}(5) = 0$

$$= \frac{1}{2}\left(\quad\right)^{-1/2}\cdot\left(\frac{d}{dx}\left(\quad\right)\right)$$

$$= \frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot\frac{d}{dx}\left(e^{-x/7}+5\right)$$

$$= \frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot\frac{d}{dx}\left(e^{-x/7}\right)$$

$$= \frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot\left(e^{\quad}\cdot\frac{d}{dx}\left(\quad\right)\right)$$

$$= \frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot\left(e^{-x/7}\cdot\frac{d}{dx}\left(-\frac{x}{7}\right)\right)$$

$$= \frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot e^{-x/7}\cdot\left(-\frac{1}{7}\right)$$

Triple chain rule:

$$\frac{d}{dx}\left(\sqrt{1+e^{\sqrt{3+x^2}}}\right) = \frac{d}{dx}\left(\left(1+e^{(3+x^2)^{1/2}}\right)^{1/2}\right)$$

$$= \frac{1}{2}\left(1+e^{(3+x^2)^{1/2}}\right)^{-1/2} \cdot \frac{d}{dx}\left(1+e^{(3+x^2)^{1/2}}\right)$$

$$= \frac{1}{2}\left(1+e^{(3+x^2)^{1/2}}\right)^{-1/2} \cdot \left(e^{(3+x^2)^{1/2}} \cdot \frac{d}{dx}\left((3+x^2)^{1/2}\right)\right)$$

$$= \frac{1}{2}\left(1+e^{(3+x^2)^{1/2}}\right)^{-1/2} \cdot e^{(3+x^2)^{1/2}} \cdot \left(\frac{1}{2}(3+x^2)^{-1/2} \cdot \frac{d}{dx}(3+x^2)\right)$$

$$= \frac{1}{2}\left(1+e^{(3+x^2)^{1/2}}\right)^{-1/2} \cdot e^{(3+x^2)^{1/2}} \cdot \frac{1}{2}(3+x^2)^{-1/2} \cdot 2x$$

$$= \frac{x \cdot e^{\sqrt{3+x^2}}}{2\sqrt{1+e^{\sqrt{3+x^2}}}\sqrt{3+x^2}}$$

It's possible to use the product + chain rules instead of the quotient rule.

Ex: $\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{d}{dx}\left(x \cdot (x^2+1)^{-1}\right)$

$f(x) = x$ $\quad g(x) = (x^2+1)^{-1}$

$f'(x) = 1$ $\quad g'(x) =$ chain rule

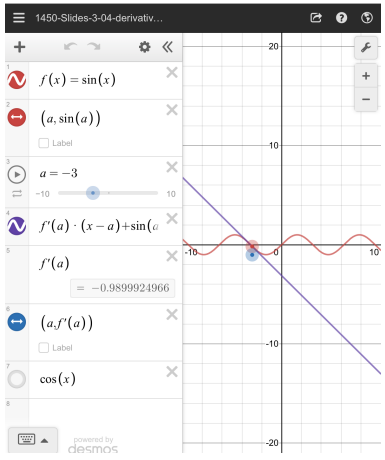
$g'(x) = -(x^2+1)^{-2} \cdot (2x)$

$= 1 \cdot (x^2+1)^{-1} + x \cdot (-(x^2+1)^{-2} \cdot 2x)$

$= \frac{1}{x^2+1} - \frac{2x^2}{(x^2+1)^2}$

Section 3.5 - The Trigonometric Functions

Goal: Figure out the derivative of $\sin(x)$, $\cos(x)$, and $\tan(x)$.



Warning: Assume we're working in radians, not degrees, forever.

Fact #1 $\frac{d}{dx}(\sin(x)) = \cos(x)$

Fact #2 $\frac{d}{dx}(\cos(x)) = -\sin(x)$

Find $\frac{d}{d\theta}\left(2 \cdot \sin(3\theta)\right)$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$= 2 \cdot \frac{d}{d\theta}(\sin(3\theta))$

$= 2 \cdot \cos(3\theta) \cdot \underbrace{\frac{d}{d\theta}(3\theta)}_{3} = 6 \cdot \cos(3\theta)$

Find $\frac{d}{dx}(\cos^2(x))$

$\cos^2(x) = (\cos(x))^2$

$= 2 \cdot \cos(x) \cdot \underbrace{\frac{d}{dx}(\cos(x))}_{-\sin(x)} = (2)(\cos(x))(-\sin(x))$

$= -2\cos(x)\sin(x)$

Ex: $\frac{d}{dx}\left(\cos(x^2)\right) =$

$$-\sin(x^2)\cdot\frac{d}{dx}(x^2)$$

$$= -2x\cdot\sin(x^2)$$

What is the derivative of $\tan(x)$?

Remember: $\tan(x) = \frac{\sin(x)}{\cos(x)}$ quotient rule!

$$\frac{d}{dx}(\tan(x)) = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$$

$$= \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

Fact 3: $\frac{d}{dx}(\tan(x)) = \frac{1}{\cos^2(x)}$

$$\left(= \sec^2(x)\right)$$