Math 1450 - Calculus 1

Wed, Oct. 8

Announcements:

* Homework 6 due Thursday night, covers 3.1, 3.2, 3.3
* Quiz 5 in discussion on Thursday, covers sugg. HW from last Fri, Mon, and today
* Exam 2 is next Wed, 5pm-6pm, this room

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 3.4: The Chain Rule

Group Work:

(1) $\dfrac{1}{1+e^x}$ $\quad \boxed{\dfrac{-e^x}{(1+e^x)^2}}$

(2) $x^5 \cdot e^x$

$\boxed{x^4 e^x (5+x)}$

(3) $(e^x + x^3) \cdot 2^x$

$(e^x + 3x^2) \cdot 2^x + (e^x + x^3) \cdot \ln(2) \cdot 2^x$

(4) $\dfrac{x^2+1}{x+1}$

$$\frac{(x+1)(2x)-(x^2+1)(1)}{(x+1)^2} = \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2}$$

(5) $\dfrac{4x}{x^2 \cdot e^x}$

$$\frac{(x^2e^x)(4)-(4x)(x^2e^x+2xe^x)}{(x^2e^x)^2} = -\frac{4x^2e^x(1+x)}{(x^2e^x)^2} = \frac{-4(1+x)}{x^2e^x}$$

Section 3.4: The Chain Rule

Examples of things we can't take the deriv. of yet

$$e^{x+x^2} \qquad (x^2+1)^{10} \qquad \sqrt{2^{(-x^3+1)}}$$

These are all <u>compositions</u> $f(g(x))$

or

$f(g(h(x)))$

$f(g(x))$

$$\xrightarrow{x} \boxed{\; g \;} \xrightarrow{g(x)} \boxed{\; f \;} \xrightarrow{f(g(x))}$$

Chain Rule: How to take the deriv. of compositions.

The Chain Rule:

$$\frac{d}{dx}\left(f(g(x))\right) = \underbrace{f'(g(x))}_{\text{take the derivative of the outer function, then plug into it the inner function (not the deriv. of the inner function)}} \cdot \underbrace{g'(x)}_{\text{then multiply by the deriv. of the inner function}}$$

take the derivative of the outer function, then plug into it the inner function (not the deriv. of the inner function)

then multiply by the deriv. of the inner function

Requires tons of practice!

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))\cdot g'(x)$$

Ex: $\frac{d}{dx}\left((x^2+1)^{100}\right)$

Step 1: Figure out what to make $f(x)$ and $g(x)$ so that $(x^2+1)^{100} = f(g(x))$.

$f(x) = x^{100}$

$g(x) = x^2+1$

$$\xrightarrow{?} \boxed{f} \xrightarrow{?^{100}}$$

$$\xrightarrow{?} \boxed{g} \xrightarrow{?^2+1}$$

$f(t) = t^{100}$

$g(t) = t^2+1$

$$\xrightarrow[x]{?} \boxed{g} \xrightarrow[x^2+1]{?^2+1} \boxed{f} \xrightarrow[(x^2+1)^{100}]{(?^2+1)^{100}}$$

Check:

$f(g(x)) = f(x^2+1) = (x^2+1)^{100}$. ✓

Different from $f(x)\cdot g(x) = x^{100}\cdot(x^2+1)$

Ex: $\frac{d}{dx}\left((x^2+1)^{100}\right)$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$f(x) = x^{100}$ $\quad$ $f'(x) = 100x^{99}$

$g(x) = x^2+1$ $\quad$ $g'(x) = 2x$

$$= f'(g(x)) \cdot g'(x)$$

$$= f'(x^2+1) \cdot g'(x)$$

$$= 100 \cdot (x^2+1)^{99} \cdot g'(x)$$

$$= 100 \cdot (x^2+1)^{99} \cdot 2x = \boxed{200x(x^2+1)^{99}}$$

Another way to think about this:

- pretend the inside function is just x and take the derivative $f'(g(x))$
- then multiply by the derivative of the inside $g'(x)$

Ex: $(x^2+1)^{100} = {}^{100} \rightsquigarrow 100 \cdot {}^{99}$

$$100 \cdot {}^{99} \cdot \frac{d}{dx} = 100(x^2+1)^{99} \cdot (2x)$$

Ex: $\frac{d}{dx}\left(e^{(x^2)}\right)$

outside : e^x

inside : x^2

$f(x) = e^x \qquad f'(x) = e^x$

$g(x) = x^2 \qquad g'(x) = 2x$

Check: $f(g(x)) = f(x^2) = e^{(x^2)}$ ✓

If we had it backward:

$f(x) = x^2, \quad g(x) = e^x$

Check: $f(g(x)) = f(e^x) = (e^x)^2$ ✗

$$f'(g(x)) \cdot g'(x) = f'(x^2) \cdot g'(x) = e^{(x^2)} \cdot g'(x)$$

$$\boxed{= e^{(x^2)} \cdot 2x = 2x e^{(x^2)}}$$

Cloud Method:

$$e^x \rightsquigarrow e^x$$

$$e^{(x^2)} = e^{\text{☁}} \xrightarrow{\text{deriv.}} e^{\text{☁}} \cdot \underbrace{\frac{d}{dx}\text{☁}}$$

The chain rule says this is how you account for ☁ not being literally just x.

$$= e^{(x^2)} \cdot \frac{d}{dx}(x^2)$$

$$\boxed{= e^{(x^2)} \cdot 2x}$$

Ex where we need the chain rule twice:

$$\left(e^{-x/7}+5\right)^{1/2} \qquad f(g(h(x)))$$

outside is $x^{1/2}$, inside is everything else

$f(x) = x^{1/2}$

$g(x) = e^{-x/7}+5$

Check: $f(g(x)) = f\left(e^{-x/7}+5\right) = \left(e^{-x/7}+5\right)^{1/2}$ ✓

$f'(x) = \frac{1}{2}x^{-1/2}$

$$g'(x) = \frac{d}{dx}\left(e^{-x/7}+5\right) = \frac{d}{dx}\left(e^{-x/7}\right) + \cancel{\frac{d}{dx}(5)}^{\,0}$$

↳ We need the chain rule.

$$\frac{d}{dx}\left(e^{-x/7}\right)$$

$f(x) = e^x$ $\quad f'(x) = e^x$

$g(x) = -x/7$ $\quad g'(x) = -1/7$

$-\frac{1}{7}\cdot x$

Check: $f(g(x)) = f(-x/7) = e^{-x/7}$ ✓

$= f'(g(x)) \cdot g'(x)$

$= f'\left(-\frac{x}{7}\right) \cdot \left(-\frac{1}{7}\right)$

$= \boxed{e^{(-x/7)} \cdot \left(-\frac{1}{7}\right)}$

$g(x) = -\frac{x}{7} = \left(-\frac{1}{7}\right) \cdot x$

Ex where we need the chain rule twice:

$$\left(e^{-x/7}+5\right)^{1/2}$$

$f(g(h(x)))$

outside is $x^{1/2}$, inside is everything else

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}$

$g(x)=e^{-x/7}+5$

$g'(x)=\left(-\frac{1}{7}\right)\cdot e^{-x/7}$

$$f'(g(x))\cdot g'(x)$$

$$\boxed{\frac{1}{2}\left(e^{-x/7}+5\right)^{-1/2}\cdot\left(-\frac{1}{7}\right)e^{-x/7}}$$