Math 1450 - Calculus 1

Mon, Oct. 6

Announcements:

* Homework 6 due Thursday night, covers 3.1, 3.2, 3.3
* Quiz 5 in discussion on Thursday, covers sugg. HW from last Fri, today, and Wed
* Don't forget about the tutoring center!

marquette.edu/tutoring

Today:

-> 3.3: The product rule and the quotient rule

Cudahy 307

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Section 3.3: The Product and Quotient Rules

In 3.1 + 3.2:

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \left(\frac{d}{dx} f(x)\right) \qquad (c \cdot f(x))' = c \cdot f'(x)$$

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$$

$$\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$$

$$\frac{d}{dx}\left(a^x\right) = \ln(a) \cdot a^x$$

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) \not= f'(x) \cdot g'(x)$$

Product Rule:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Another way to write this:

$$\frac{d}{dx}(u\cdot v) = \frac{du}{dx}\cdot v + u\cdot\frac{dv}{dx}$$

$u \to f$, $v \to g$, $\frac{du}{dx} \to f'$, $v \to g$, $u \to f$, $\frac{dv}{dx} \to g'$

"(derivative of the first) times the second plus first times (derivative of the second)"

Ex: Find the derivative of $x^2 \cdot e^x$

$f(x)$ $g(x)$

$$\frac{d}{dx}(x^2 \cdot e^x) = \frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$f'(x) = 2x$ $\quad g'(x) = \ln(e) \cdot e^x = e^x$

$$= (2x)(e^x) + (x^2)(e^x)$$

$$= \boxed{x \cdot e^x \cdot (2 + x).}$$

Ex: Find the derivative of $\underbrace{(7x^3+2x+1)}_{f(x)} \cdot \underbrace{3^x}_{g(x)}$.

$f'(x) = 21x^2+2$ $\qquad$ $g'(x) = \ln(3) \cdot 3^x$

$$f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= (21x^2+2) \cdot 3^x + (7x^3+2x+1) \cdot \ln(3) \cdot 3^x$$

$$= 3^x \cdot \left(21x^2+2 + \ln(3) \cdot (7x^3+2x+1) \right)$$

Ex: We can do some fractions with the product rule.

$$\frac{5^x}{x^2} = (5^x)(x^{-2})$$

$$\frac{d}{dx}\left(\frac{5^x}{x^2}\right) = \frac{d}{dx}\left(\underbrace{(5^x)}_{f(x)}\underbrace{(x^{-2})}_{g(x)}\right)$$

$$\frac{d}{dx}(x^{-2}) = -2x^{-2-1} = -2x^{-3}$$

$$= (\ln(5)\cdot 5^x)\cdot(x^{-2}) + 5^x\cdot(-2)(x^{-3})$$

$$= \frac{\ln(5)\cdot 5^x}{x^2} - \frac{2\cdot 5^x}{x^3}$$

$$= \frac{5^x}{x^2}\cdot\left(\ln(5) - \frac{2}{x}\right)$$

Ex of one we can't turn into a product yet:

$$\frac{5^x}{x^3+1} = \underbrace{5^x}_{f} \cdot \underbrace{(x^3+1)^{-1}}_{g}$$

we don't know how to find g' yet

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(\frac{du}{dx}\right)\cdot v - u\cdot\left(\frac{dv}{dx}\right)}{v^2}$$

low = denom = $g(x)$
high = numer = $f(x)$

"$\underbrace{\text{low}}_{g(x)}\ \underbrace{\text{d-high}}_{f'(x)}$ minus $\underbrace{\text{high}}_{f(x)}\ \underbrace{\text{d-low}}_{g'(x)}$ over $\underbrace{\text{low}\cdot\text{low}}_{(g(x))^2}$"

Ex: Find the derivative of $\frac{3x^2+1}{x^3+5}$.

$f(x) = 3x^2+1$

$g(x) = x^3+5$

$f'(x) = 6x$ $\quad g'(x) = 3x^2$

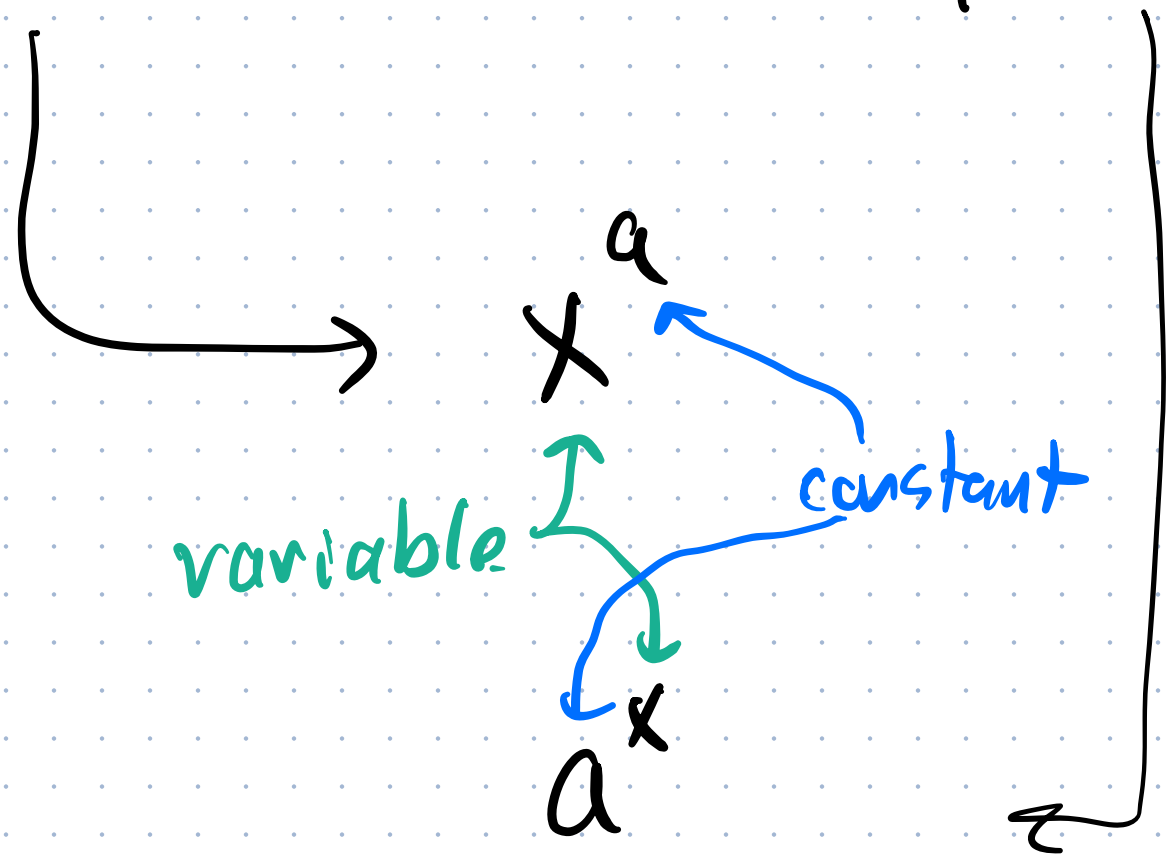
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$= \frac{(x^3+5)(6x) - (3x^2+1)(3x^2)}{(x^3+5)^2}$$

$$= \frac{(6x^4+30x) - (9x^4+3x^2)}{(x^3+5)^2} = \frac{-3x^4-3x^2+30x}{(x^3+5)^2}$$

$$= \frac{3x(-x^3-x+10)}{(x^3+5)^2}$$

Power rule vs. exponential rule



x^5 $\quad$ 5^x

Group Work:

(1) $\frac{1}{1+e^x}$ $\boxed{\frac{-e^x}{(1+e^x)^2}}$

(2) $x^5 \cdot e^x$

$\boxed{x^4 e^x (5+x)}$

(3) $(e^x + x^3) \cdot 2^x$

(4) $\frac{x^2+1}{x+1}$

(5) $\frac{4x}{x^2 \cdot e^x}$