Math 1450 – Calculus 1

~~Wed~~, Oct. ~~1~~
Fri 3

Announcements:

* Homework 6 due Thursday night, covers 3.1, 3.2, 3.3
* Quiz 5 in discussion on Thursday, covers sugg. HW from today, next Mon, next Wed
* Don't forget about the tutoring center!

  marquette.edu/tutoring

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Today:

→ 3.1: Powers and Polynomials
→ 3.2: Exponential Functions

Theorem 3.1: The derivative of $c \cdot f(x)$ is $c \cdot f'(x)$.

$\frac{dy}{dx}$

Other notations:

$$(c \cdot f(x))' = c \cdot f'(x)$$

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot f'(x)$$

"take the derivative of"

Topic 2: Sums + Differences of Functions

Let $f(x)$ and $g(x)$ be two functions. What is the derivative of $f(x)+g(x)$?

Theorem 3.2: $\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x)+g'(x)$

To find the deriv. of $f+g$, take the individual derivatives of f and g and add them together.

Differences: $\frac{d}{dx}\left(f(x)-g(x)\right) = f'(x)-g'(x)$

Warning

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) \neq f'(x)\cdot g'(x)$$

Topic 3: Power Rule

constant, any number

Theorem: $$\frac{d}{dx}\left(x^{n}\right) = n \cdot x^{n-1}$$

variable

Ex: $\left(x^{3}\right)' = 3 \cdot x^{3-1} = 3x^{2}$

wrong

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) \cancel{\neq \frac{1}{\frac{1}{2}x^{-1/2}} = 2\cdot\sqrt{x}}$$

$$= \frac{d}{dx}\left(x^{-1/2}\right) = \left(-\frac{1}{2}\right)\cdot x^{(-1/2)-1} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$$

$$\frac{d}{dx}\left(5\cdot\sqrt{x}-\frac{10}{x^2}+\frac{1}{2\sqrt{x}}\right)$$

Sum rule

$$\boxed{\frac{1}{2}-1=\frac{1}{2}-\frac{2}{2}=\frac{1-2}{2}=-\frac{1}{2}}$$

$$=\frac{d}{dx}(5\cdot\sqrt{x})-\frac{d}{dx}\left(\frac{10}{x^2}\right)+\frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)$$

constant mult

$$=5\cdot\frac{d}{dx}(\sqrt{x})-10\frac{d}{dx}\left(\frac{1}{x^2}\right)+\frac{1}{2}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

rewrite as powers

$$=5\cdot\frac{d}{dx}\left(x^{1/2}\right)-10\cdot\frac{d}{dx}\left(x^{-2}\right)+\frac{1}{2}\cdot\frac{d}{dx}\left(x^{-1/2}\right)$$

$$=5\cdot\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}-10\cdot(-2)\cdot x^{-2-1}+\frac{1}{2}\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

$$=\underline{\frac{5}{2}x^{-1/2}+20x^{-3}-\frac{1}{4}x^{-3/2}}=\underline{\frac{5}{2\sqrt{x}}+\frac{20}{x^3}-\frac{1}{4x^{3/2}}}$$

Warning

The power rule does not work for exponential functions.

$$\cancel{\frac{d}{dx}\left(3^x\right) \neq x \cdot 3^{x-1}}$$

3.2: Exponential Functions

In 3.1, we saw derivatives of power functions and polynomials.

Derivative of a degree 4 polynomial
= degree 3 polynomial

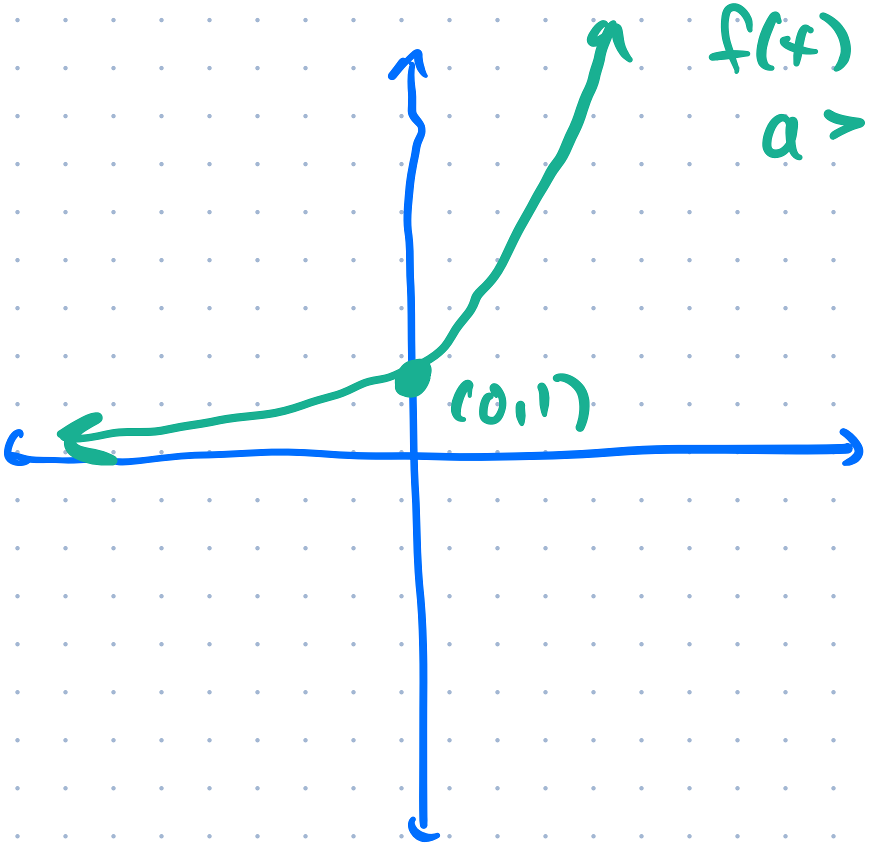
Deriv. of a degree 3 poly = degree 2 poly

Deriv. of a degree 2 poly = degree 1 poly
= line

deriv. of a line = constant

deriv. of a constant = 0.

What is the derivative of an exponential function?



$f(x) = a^x$

$a > 1$

Facts about f':

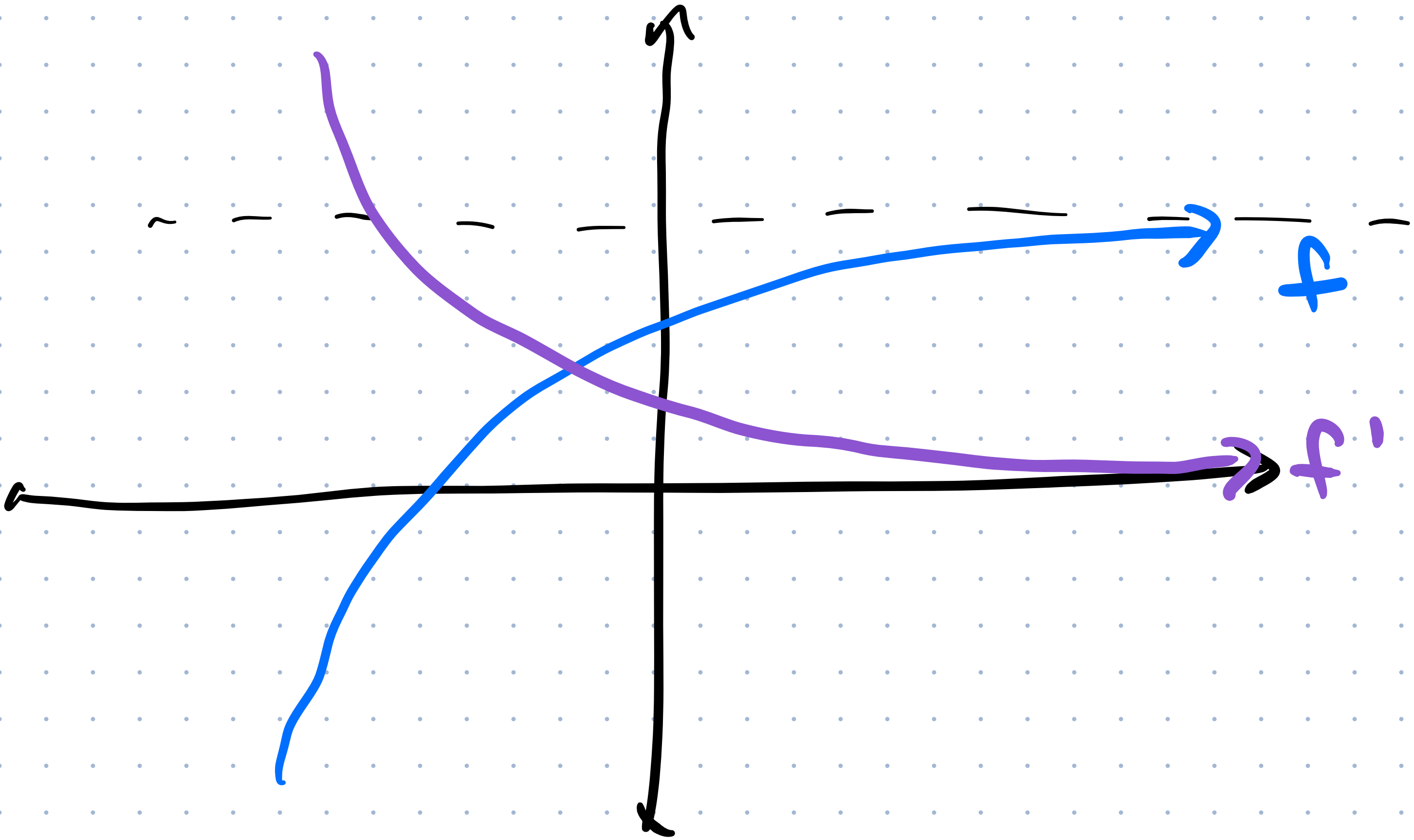
* $f' > 0$ everywhere
* f' is increasing everywhere
* $f' \to 0$ as $x \to -\infty$
* $f' \to \infty$ very quickly as $x \to \infty$

Does this sound like a function we know?

f' increasing vs $\boxed{f' > 0}$

$\Downarrow$ going up

$\Downarrow$ f is increasing



What is the derivative of $g(x) = 2^x$?

$2^x \cdot 2^h$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h}$$

$$= \lim_{h \to 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \to 0} \frac{2^x \cdot (2^h - 1)}{h}$$

$$= \lim_{h \to 0} \left[2^x \cdot \frac{2^h - 1}{h} \right] = \underbrace{\left[\lim_{h \to 0} 2^x \right]}_{2^x} \cdot \left[\lim_{h \to 0} \frac{2^h - 1}{h} \right]$$

$$= 2^x \cdot \underbrace{\lim_{h \to 0} \left[\frac{2^h - 1}{h} \right]}_{\text{some \#}}$$

$\approx 0.693\ldots\ldots = \ln(2)$

$$\frac{d}{dx}(2^x) = \underbrace{\ln(2)}_{\text{a \#}} \cdot 2^x$$

Exponential Derivative Formula

$$\frac{d}{dx}\left(a^x\right) = \underbrace{\ln(a)}_{\text{a constant multiplier}} \cdot a^x$$

exponential function

a constant multiplier

Ex:

$$\frac{d}{dx}\left(e^x\right) = \ln(e) \cdot e^x = 1 \cdot e^x = e^x$$

$$\frac{d}{dx}\left(\left(\tfrac{1}{2}\right)^x\right) = \underbrace{\ln\left(\tfrac{1}{2}\right)}_{\text{negative \#}} \cdot \left(\tfrac{1}{2}\right)^x$$

negative #

≈ -0.693

Ex: Find the derivative of

$$2\cdot 3^x + 5\cdot e^x$$

$$\frac{d}{dx}\left(2\cdot 3^x + 5\cdot e^x\right) = \frac{d}{dx}\left(2\cdot 3^x\right) + \frac{d}{dx}\left(5\cdot e^x\right)$$

$$= 2\cdot \frac{d}{dx}\left(3^x\right) + \boxed{5\cdot \frac{d}{dx}\left(e^x\right)}$$

$$= 2\cdot\left(\ln(3)\cdot 3^x\right) + 5\cdot\left(\cancel{\ln(e)\cdot}\, e^x\right)$$

$$= \underline{2\ln(3)\, 3^x + 5\, e^x}$$

Ex: Find the derivative of

$$\boxed{2\cdot 3^x + 5\cdot e^x}$$

different notation

$$(2\cdot 3^x + 5\cdot e^x)' = (2\cdot 3^x)' + (5\cdot e^x)'$$

$$= 2\cdot (3^x)' + 5\cdot (e^x)'$$

$$= 2\cdot\left(\ln(3)\cdot 3^x\right) + 5\cdot\left(\cancel{\ln(e)}\cdot e^x\right)$$

$$= \boxed{2\cdot\ln(3)\cdot 3^x + 5\cdot e^x}$$

What is the 100th derivative of $5e^x$

$5e^x$

Group Work:

Find each derivative

#9] $\frac{3^x}{3} + \frac{33}{\sqrt{x}}$

$33x^{-1/2}$

$\frac{1}{3}\ln(3)\cdot 3^x - \frac{33}{2}x^{-3/2}$

#17] $e^{\pi} + \pi^x$

$0 + \ln(\pi)\cdot \pi^x$