Math 1450 - Calculus 1

Wed, Oct. 1

Announcements:

* Homework 5 due Thursday night, covers 2.3, 2.4, 2.5, 2.6
* Quiz 4 in discussion on Thursday, covers sugg. HW from last Fri, Mon, and today
* Don't forget about the tutoring center!

marquette.edu/tutoring

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!
12-1 on Wed

Today:

→ 2.6: Differentiability
→ 3.1: Powers and Polynomials

Section 2.6 - Differentiability

We say that a function $f(x)$ is __differentiable__ at a point $x=a$ if:

* the derivative exists at $x=a$

or, rephrased

* $\boxed{\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$ exists

the derivative at $x=a$

What could make a function <u>NOT</u> differentiable?

In other words, what could make $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ not exist?

1) if $f(a)$ doesn't exist

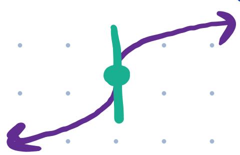
$\frac{1}{x}$ is not <u>diff.</u> at $x=0$

2) if f isn't continuous at $x=a$ (we'll justify this later)

(not continuous) implies (not diff.)

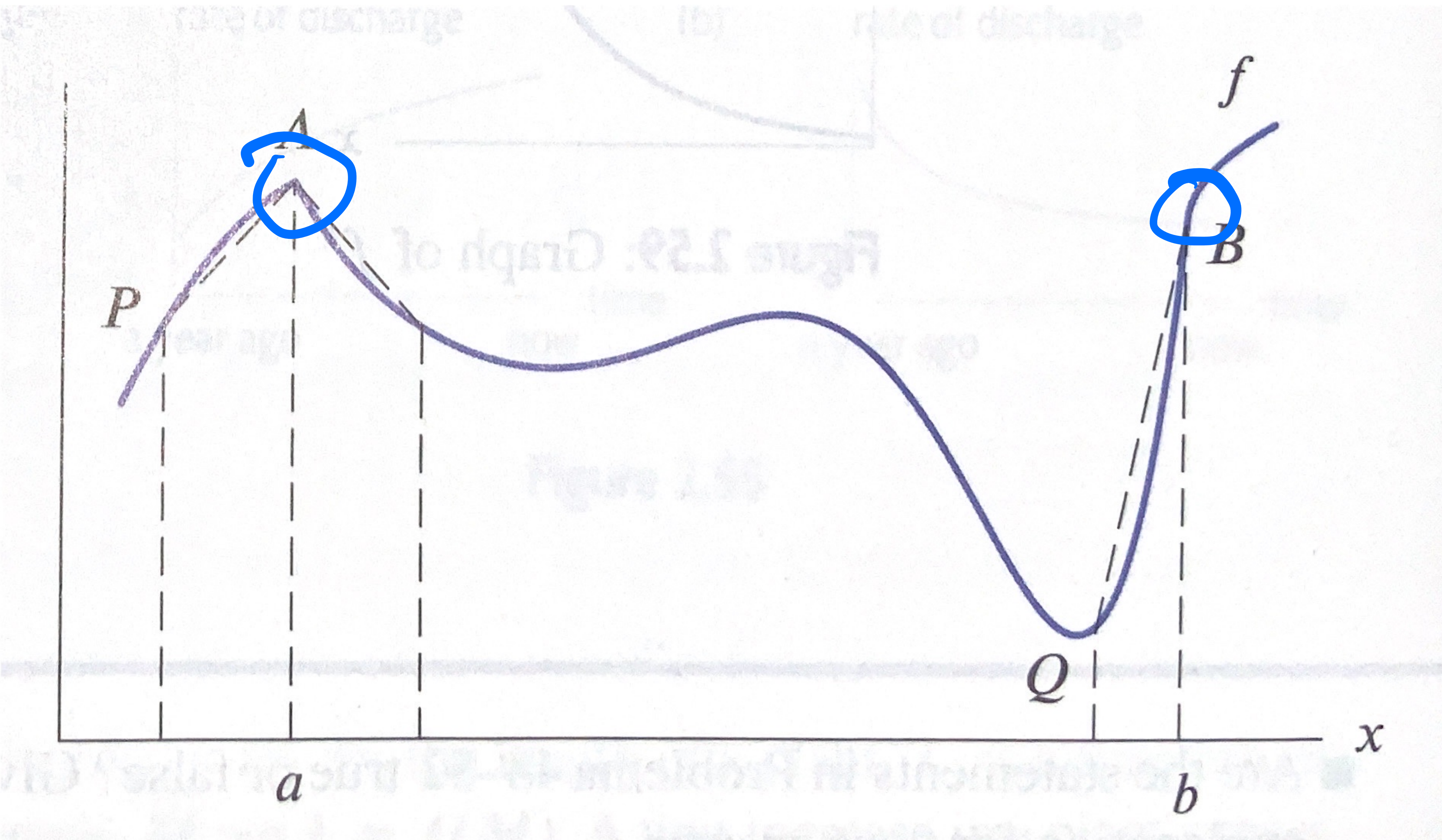
3) f has a sharp corner at $x=a$ (not smooth)

4) f has a vertical tangent line at $x=a$

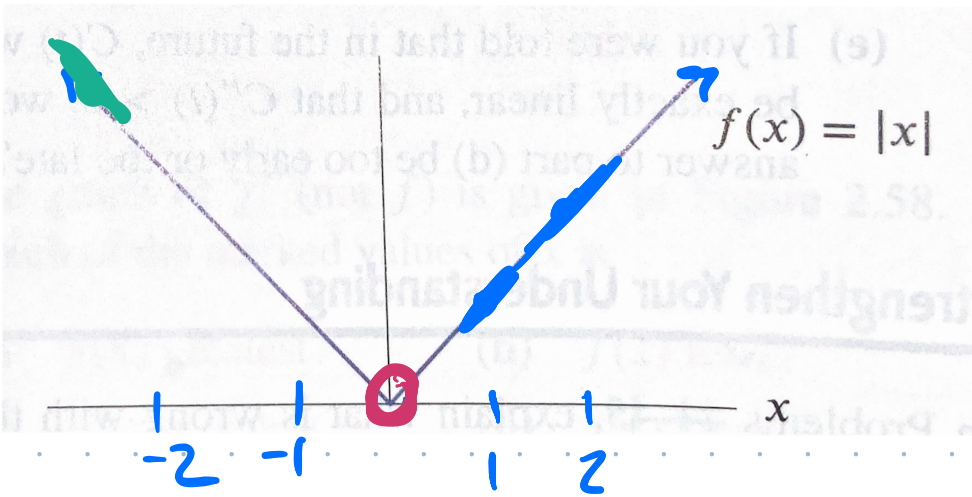


$f(x) = \sqrt[3]{x}$

Ex: Where is the function not differentiable?



Ex: Where is the function not differentiable?



$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$f'(3) = 1$

$f'(2) = 1$

$f'(1) = 1$

$f'(0.0001) = 1$

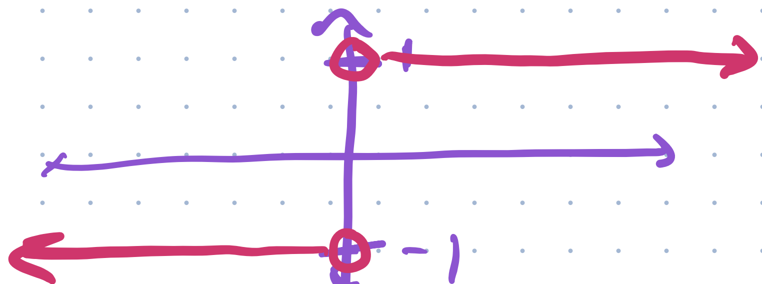
$f'(-3) = -1$

$f'(-2) = -1$

$f'(-1) = -1$

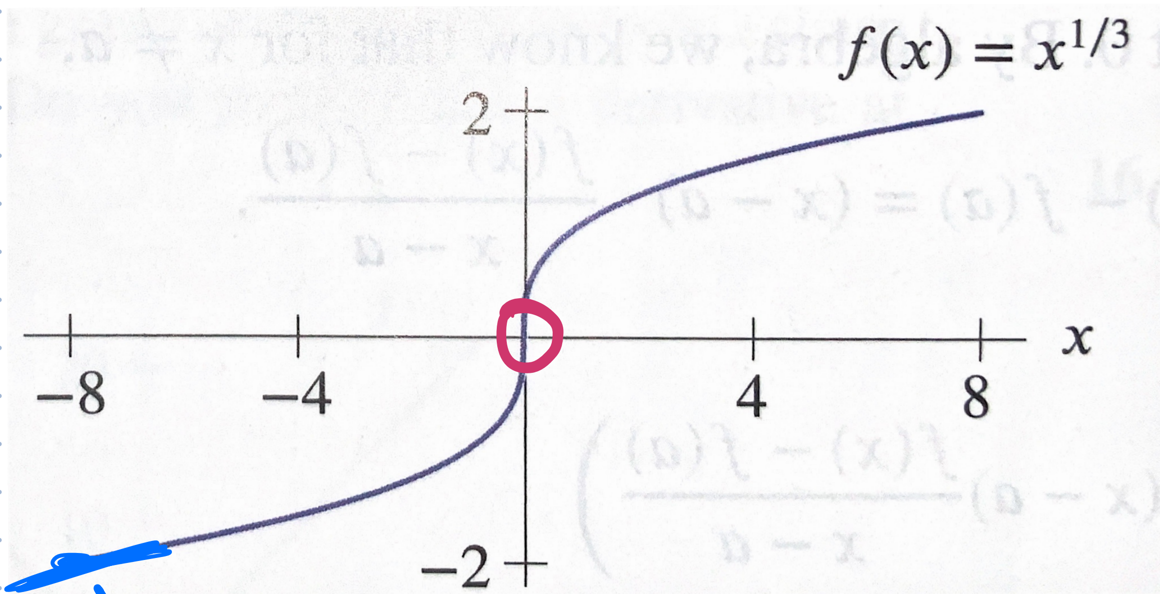
$f'(-0.0001) = -1$

Graph of f':



Ex:

$f(x) = \sqrt[3]{x}$ not diff. at $x = 0$



$\frac{1}{12}$

Power rule in 3.1: The derivative of x^a is $a \cdot x^{a-1}$

$$f'(x) = \frac{1}{3} \cdot x^{\left(\frac{1}{3} - 1\right)} = \frac{1}{3} x^{-2/3} = \frac{1}{3 \cdot x^{2/3}}$$

$$f'(-8) = \frac{1}{3 \cdot (-8)^{2/3}} = \frac{1}{12}$$

$((-8)^{1/3})^2$

$$f'(0) = \frac{1}{3 \cdot 0^{2/3}} = \frac{1}{0} \text{ undefined}$$

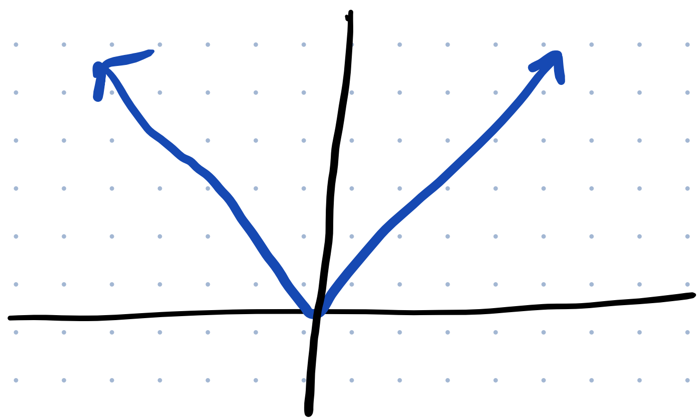
Differentiability vs. Continuity

diff $\Rightarrow$ continuous

mathematical fact

Theorem: If $f(x)$ is differentiable at $x=a$ then $f(x)$ is continuous at $x=a$.

Not true in reverse!



Possible to be continuous but not diff'ble

~~continuous $\Rightarrow$ diff~~

Proof in book! Simple arithmetic and use of limit properties.

Suggested HW: 1, 2, 3, 4, 5, 6, 7, 9, 11, 13

Section 3.1: Powers and Polynomials

3.1–3.5 are all about proving formulas for computing derivatives without limits

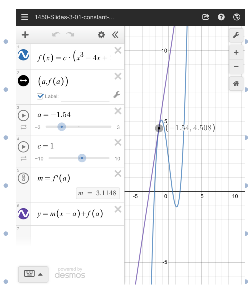
$f(x) = x^2$

$f(x) = x^{100}$

$f'(x) = 100x^{99}$

Topic 1: Constant Multiples

Let $f(x)$ be a function and $f'(x)$ its derivative. What is the derivative of $c \cdot f(x)$ if c is a constant?



Theorem 3.1: The derivative of $c \cdot f(x)$ is $c \cdot f'(x)$.

$$\frac{dy}{dx}$$

Other notations:

$$(c \cdot f(x))' = c \cdot f'(x)$$

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot f'(x)$$

"take the derivative of"

Why is this true?

Let $f(x)$ be a function and let $g(x) = c \cdot f(x)$.

Goal: $g'(x) = c \cdot f'(x)$.

By the def. of a derivative

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{(c \cdot f(x+h)) - (c \cdot f(x))}{h}$$

$$= \lim_{h \to 0} \left[c \cdot \left(\frac{f(x+h) - f(x)}{h} \right) \right]$$

property of limits

$$= \underbrace{\left[\lim_{h \to 0} c \right]}_{c} \cdot \underbrace{\left[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right]}_{f'(x)}$$

$$= c \cdot f'(x)$$

Topic 2: Sums + Differences of Functions

Let $f(x)$ and $g(x)$ be two functions. What is the derivative of $f(x)+g(x)$?

Theorem 3.2: $\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x)+g'(x)$

To find the deriv. of $f+g$, take the individual derivatives of f and g and add them together.

Differences: $\frac{d}{dx}\left(f(x)-g(x)\right) = f'(x)-g'(x)$

Warning

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) \neq f'(x)\cdot g'(x)$$

Topic 3: Power Rule

constant, any number

Theorem: $$\frac{d}{dx}\left(x^n\right) = n \cdot x^{n-1}$$

variable

Ex: $(x^3)' = 3 \cdot x^{3-1} = 3x^2$

wrong

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) \cancel{\neq \frac{1}{\frac{1}{2}x^{-1/2}} = 2 \cdot \sqrt{x}}$$

$$= \frac{d}{dx}\left(x^{-1/2}\right) = \left(-\frac{1}{2}\right) \cdot x^{(-1/2)-1} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$$

Combining these: derivatives of polynomials

sums and diffs rule

$$\frac{d}{dx}\left(5x^2+3x+2\right) = \underbrace{\frac{d}{dx}(5x^2)} + \underbrace{\frac{d}{dx}(3x)} + \underbrace{\frac{d}{dx}(2)}$$

$$= 5\cdot\frac{d}{dx}(x^2) + 3\cdot\frac{d}{dx}(x) + 0$$

$$= 5\cdot(2x) + 3\cdot(1)$$

$$= 10x+3$$

$$\frac{d}{dx}(x^1) = 1\cdot x^0$$
$$= 1$$

$$\frac{d}{dx}\left(5\cdot\sqrt{x} - \frac{10}{x^2} + \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{d}{dx}(5\cdot\sqrt{x}) - \frac{d}{dx}\left(\frac{10}{x^2}\right) + \frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)$$

$$= 5\cdot\frac{d}{dx}(\sqrt{x}) - 10\,\frac{d}{dx}\left(\frac{1}{x^2}\right) + \frac{1}{2}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$= 5\cdot\frac{d}{dx}\left(x^{1/2}\right) - 10\cdot\frac{d}{dx}(x^{-2}) + \frac{1}{2}\cdot\frac{d}{dx}\left(x^{-1/2}\right)$$