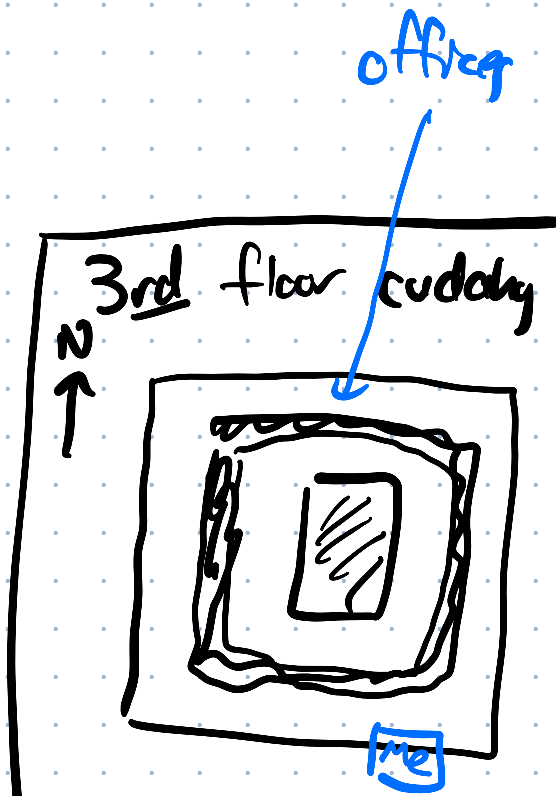
Math 1450 - Calculus 1

Friday, Sept. 26

Announcements:

* Reminder: Old videos from Fall 2020 are posted on the course calendar - lecture/exercises
* HW 5, Q4, next Thursday
* Don't forget about the tutoring center!

  marquette.edu/tutoring



Office Hours 307

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Today:

→ 2.4: Interpretations of the Derivative
→ 2.5: The Second Derivative

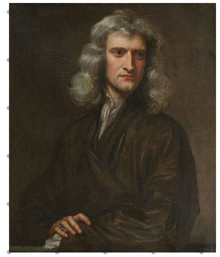
Section 2.4 – Interpretations of the Derivative

Calculus was invented in the 1600s nearly simultaneously by two people: Newton and Leibniz

They had very different viewpoints!
We mostly teach Newton's version these days, but not his notation!

Notations for a derivative:

Newton – $\dot{y}$ or $\frac{\dot{y}}{\dot{x}}$ (1704)



Lagrange – $f'(x)$ (1770)

Euler – $(Df)(x)$ (1700s)

Leibniz – $\frac{dy}{dx}$ (1680s)

Leibniz's notation can be pretty useful, especially in later courses.

Let $y = f(x)$ be a function.

$$\underbrace{\frac{dy}{dx}}_{\text{Leibniz}} = \underbrace{f'(x)}_{\text{Lagrange}}$$

slope of the T.L of $f(x)$ at a certain x-value

"d" = "difference" = Δ

think of this like a ratio

$$\frac{\text{small change in } y}{\text{small change in } x}$$

= when x changes a little bit, how much does y change?

= when x changes a little bit, how much does y change?

$f'(x) = 2$

<u>Ex:</u> $\frac{dy}{dx} = 2$ means if x increases by 0.1 then y increases by ≈ 0.2.

Leibniz notation also makes units make sense.

Suppose $s = f(t)$ measures the position of a car, in meters, after $\underbrace{t \text{ seconds.}}$ $\underline{s \text{ meters}}$

meters

$$\frac{ds}{dt} \Bigg\} \frac{\text{meters}}{\text{sec}}$$

seconds

unit of measure for $f'(t)$ which is the same as $\frac{ds}{dt}$

$5 \ ^{m}/sec$

The book uses notation

$$\underbrace{\left.\frac{ds}{dt}\right]_{t=2}}_{f'(2)}$$

to mean this ratio at $t=2$

<u>Ex:</u> The cost of extracting T tons of ore from a copper mine is $C = f(T)$ dollars. What does it mean to say $f'(2000) = 100$?

$f(2000) = 1{,}000{,}000$

→ The cost of extracting 2000 tons of ore is \$1,000,000.

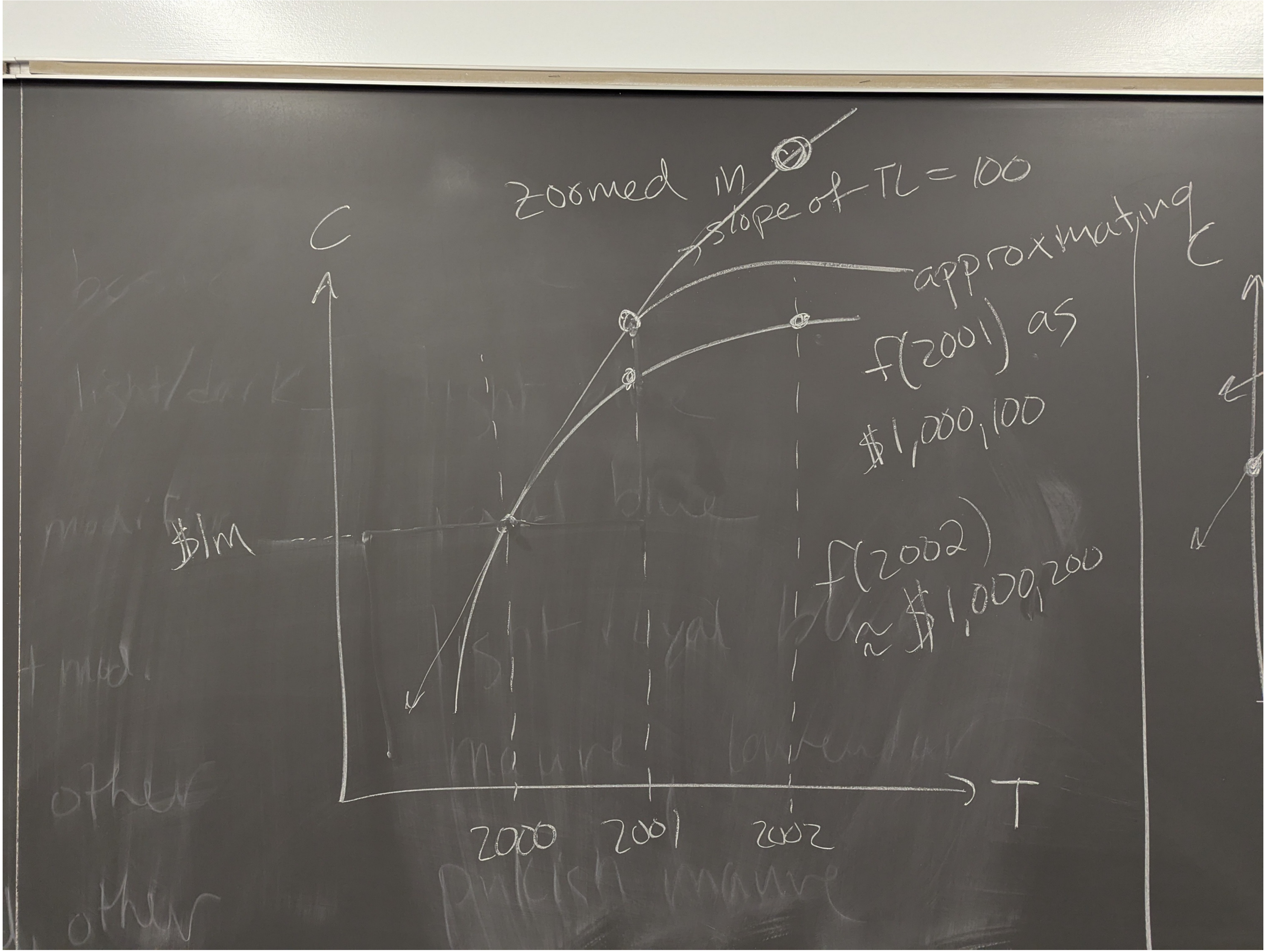
$$f'(2000) = \left.\frac{dC}{dT}\right|_{T=2000} = 100 \ \frac{\text{dollars}}{\text{ton}}$$

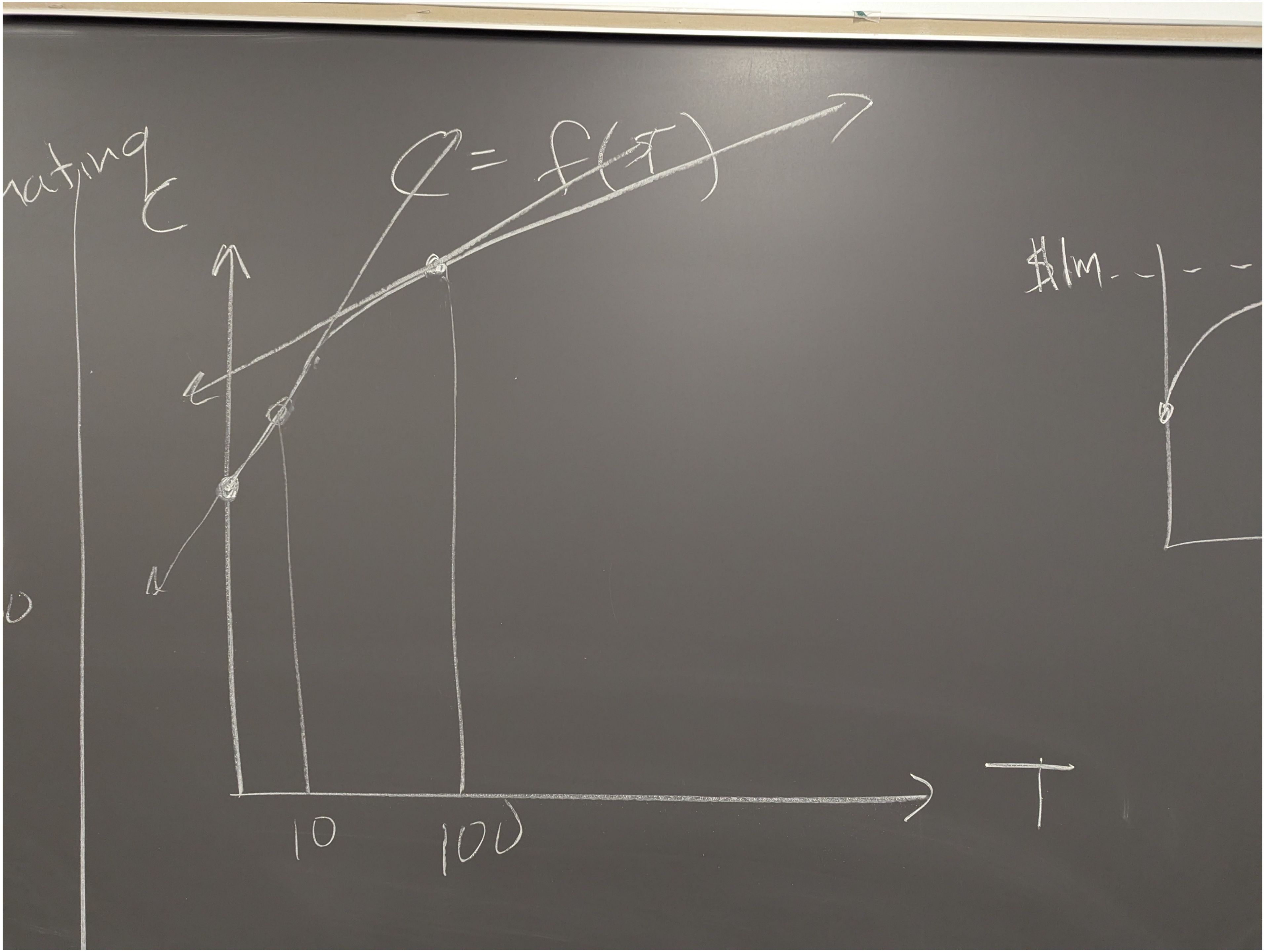
\$

tons

"When 2000 tons have already been extracted, the additional cost to extract the next ton is ≈ 100 \$/ton."

The cost to extract the next ΔT tons, is $\approx 100 \cdot \Delta T$.


zoomed in
slope of TL = 100
C
approximating
f(2001) as
$1,000,100
$1m
f(2002)
≈ $1,000,200
T
2000
2001
2002
C


C
C = f(T)
10
100
T
$1m

Ex: If $q=f(p)$ is the # of pounds of sugar produced by a manufacturer when the price per pound is p cents, then what are the units and meaning of the statement $f'(30)=50$.

$f(30)=1000$.

When the price per pound is 30 ¢, the amount produced is 1000 pounds.

p: price per pound, ¢

q: # of pounds produced, pounds

$$f'(30)=\left.\frac{dq}{dp}\right]_{p=30}=50$$

pounds / cent

$\dfrac{\text{pounds produced}}{\text{cents per pound}}$

When the price per pound is 30 ¢, the # of pounds produced is increasing at a rate of 50 pounds per 1 ¢ increase in price.

$f(30) = 1000.$

When the price per pound is 30 ¢, the amount produced is 1000 pounds.

p: price per pound, ¢

q: # of pounds produced, pounds

$$f'(30) = \left.\frac{dq}{dp}\right]_{p=30} = 50 \quad \frac{\text{pounds}}{\text{cent}}$$

$\frac{\text{pounds produced}}{\text{cents per pound}}$

When the price per pound is 30 ¢, the # of pounds produced is increasing at a rate of 50 pounds per 1 ¢ increase in price.

Approximate $f(31). \approx 1050$

$f(32) \approx 1100$

$f(40) \approx 1500$

$f(29) \approx 950.$

Suggested HW: 1, 2, 4, 5, 9, 13, 14, 15, 23, 24, 33, 34, 35, 37

Section 2.5: The Second Derivative

The derivative of $f(x)$ is $f'(x)$.

$f'(x)$ is itself a function

So, we can take the derivative of $f'(x)$.

We call this $f''(x)$, "f double prime"

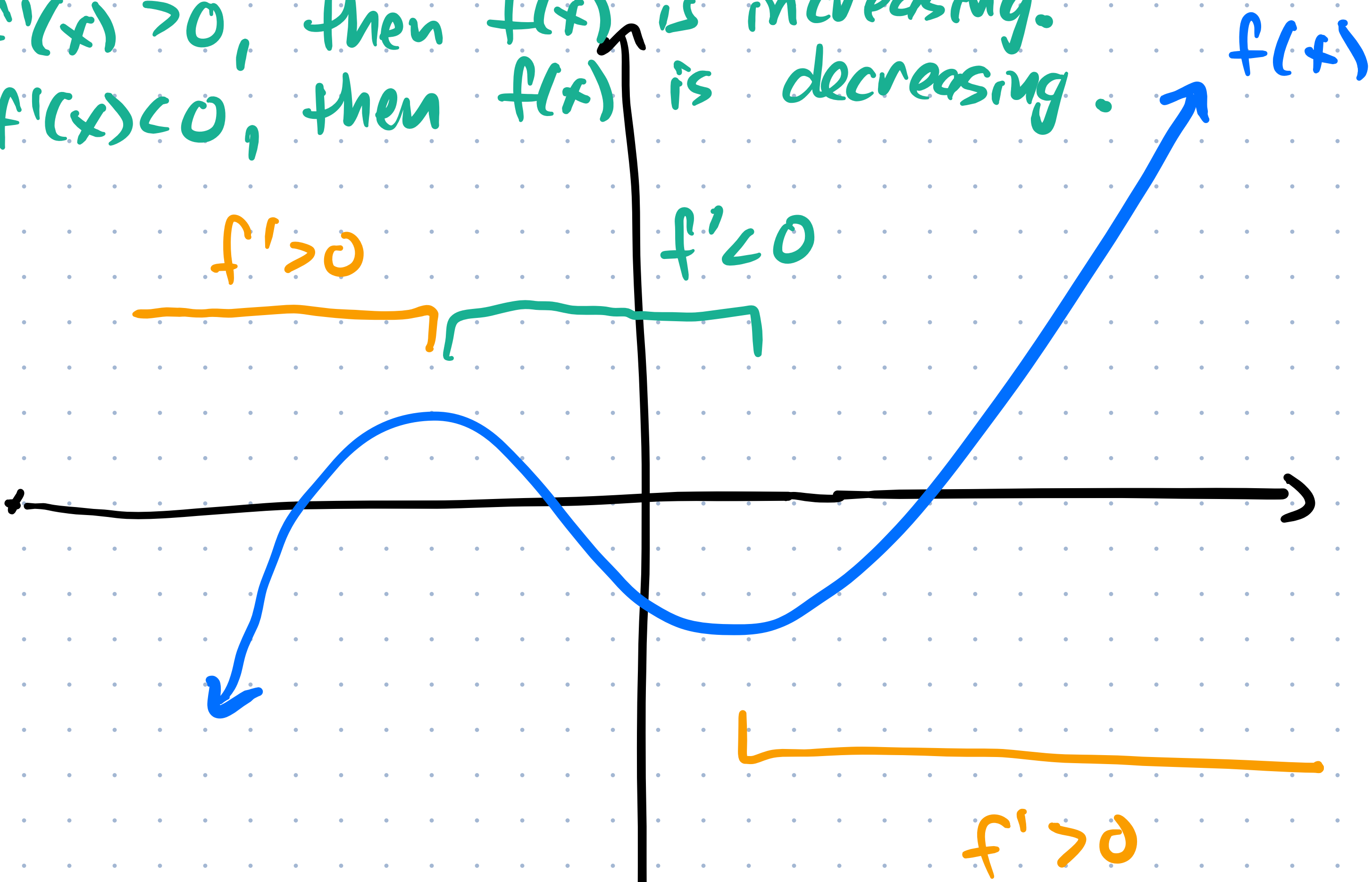
and it's the "second derivative of $f(x)$"

Third deriv: $f'''(x)$

Fourth deriv: $f^{(4)}(x)$ and so on

What does the second derivative mean?

If $f'(x) > 0$, then $f(x)$ is increasing.
If $f'(x) < 0$, then $f(x)$ is decreasing.



If $f'' > 0$, then f' is increasing, so f is getting steeper and steeper.