Math 1450 - Calculus 1

Wed, Sept. 24

Announcements:

* Quiz 3 tomorrow / covers sugg. HW from 2.1, 2.2, 2.3
* Homework 4 due tomorrow
  covers 2.1 and 2.2
* Reminder: Old videos from Fall 2020 are posted on the course calendar - lecture/exercises

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Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 2.3: The Derivative Function

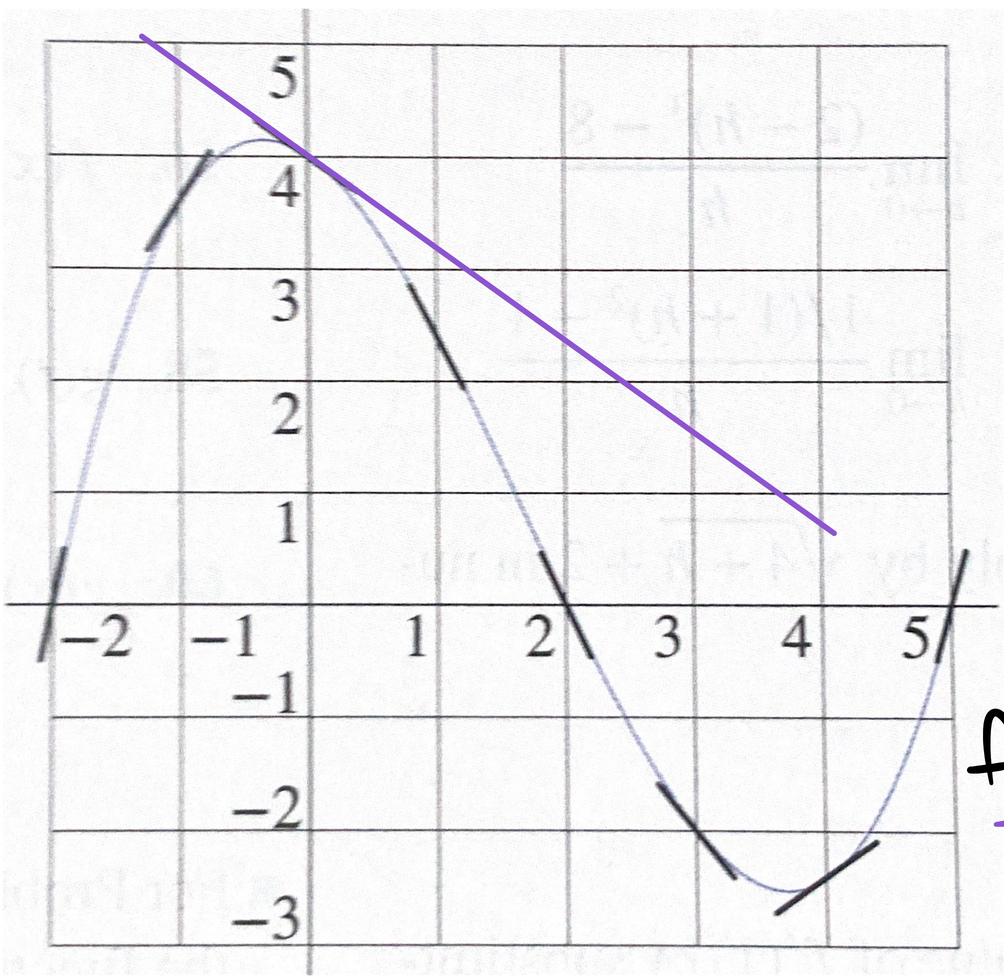
→ 2.4: Interpretations of the Derivative

Section 2.3 – The derivative function

(Same idea as Section 2.2, in more detail)

Ex: Estimate $f'(x)$ at $x = -2, -1, 0, 1, 2, 3, 4, 5$.

$f'(-2)$



$f(x)$

x	$f(x)$	$f'(x)$
-2	0	5
-1	3.5	2
0	4	-0.8
1	2.5	-2.5
2	0	-2.5
3	-2	-1
4	-2.5	0.8
5	0	5

The <u>derivative function</u>

Let $f(x)$ be some function. Its <u>derivative</u> is a new function called $f'(x)$ that takes as input some x-value and outputs the <u>slope</u> (of the T.L.) of $f(x)$ at that x-value.

$$a \longrightarrow \boxed{\;f\;} \longrightarrow f(a)$$

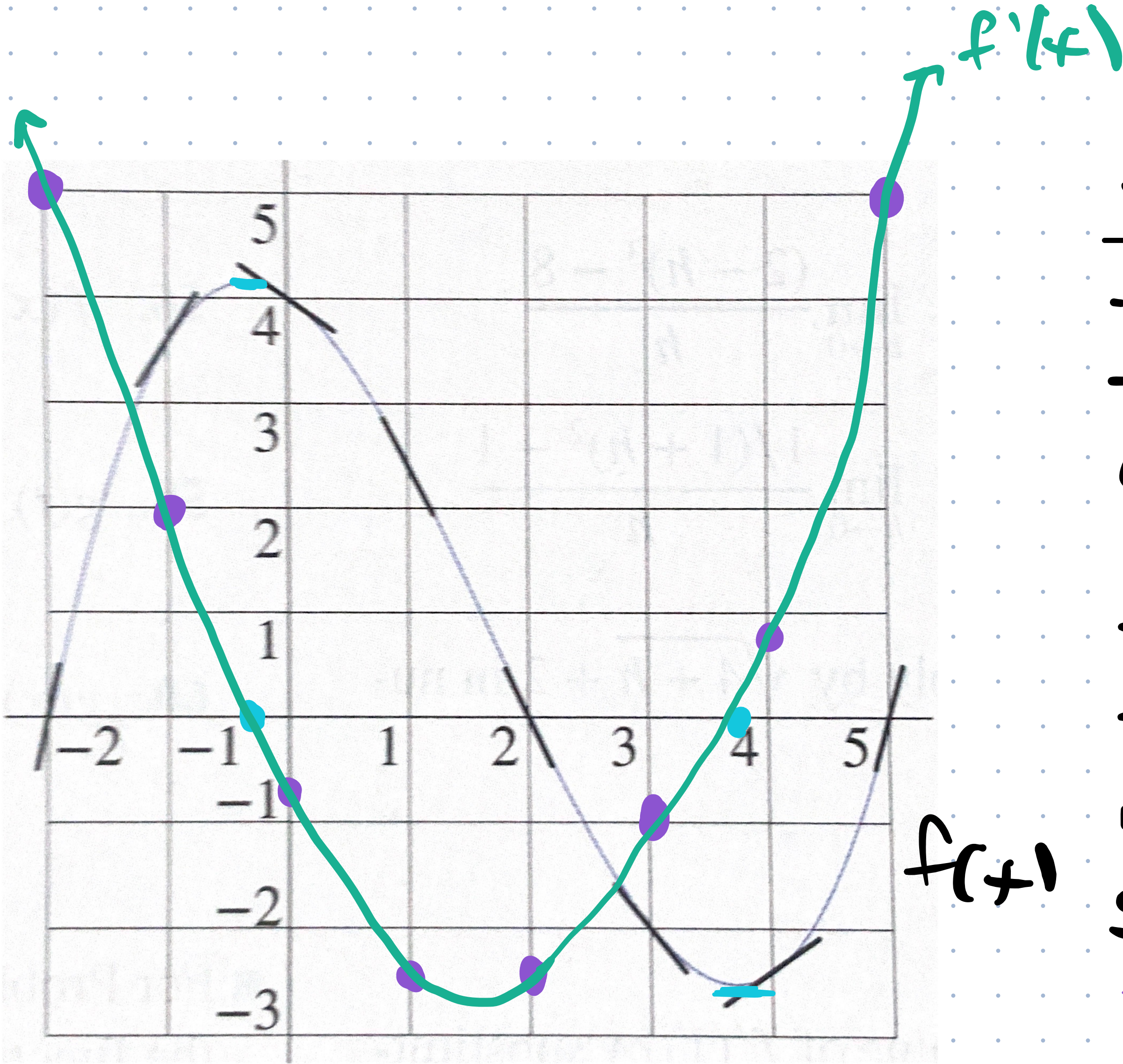
$$a \longrightarrow \boxed{\;f'\;} \longrightarrow \left[\begin{array}{l}\text{slope of } f(x)\\ \text{at } x=a\end{array}\right] = f'(a)$$

Limit Definition

$$f'(\overset{a}{x}) = \lim_{h \to 0} \frac{f(\overset{a}{x}+h) - f(\overset{a}{x})}{h}$$

Ex: On the graph below, draw the function $f'(x)$.

● = pts where $f' = 0$

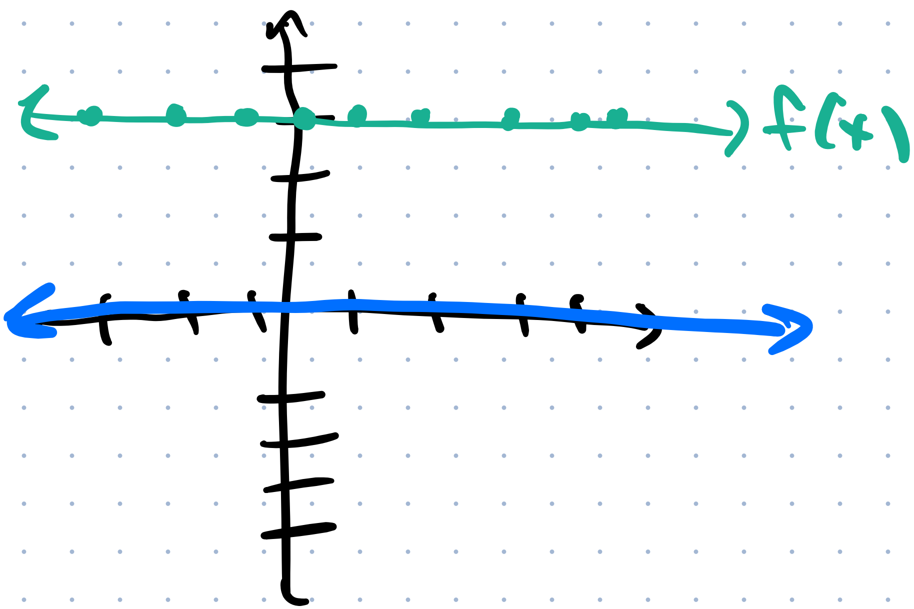


x	f(x)	f'(x)
-2	0	5
-1	3.5	2
0	4	-0.8
1	2.5	-2.5
2	0	-2.5
3	-2	-1
4	-2.5	0.8
5	0	5

First Derivative Formulas

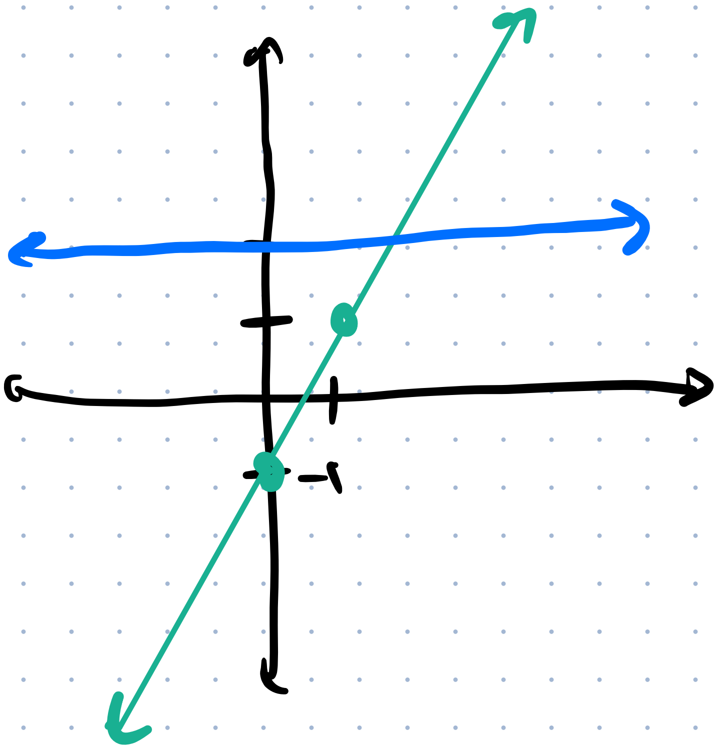
* Let $f(x)$ be a constant function.

$f(x) = k$ $\quad (f(x) = 3)$



$f'(x) = 0$

* Let $f(x)$ be any line: $f(x) = m \cdot x + b$

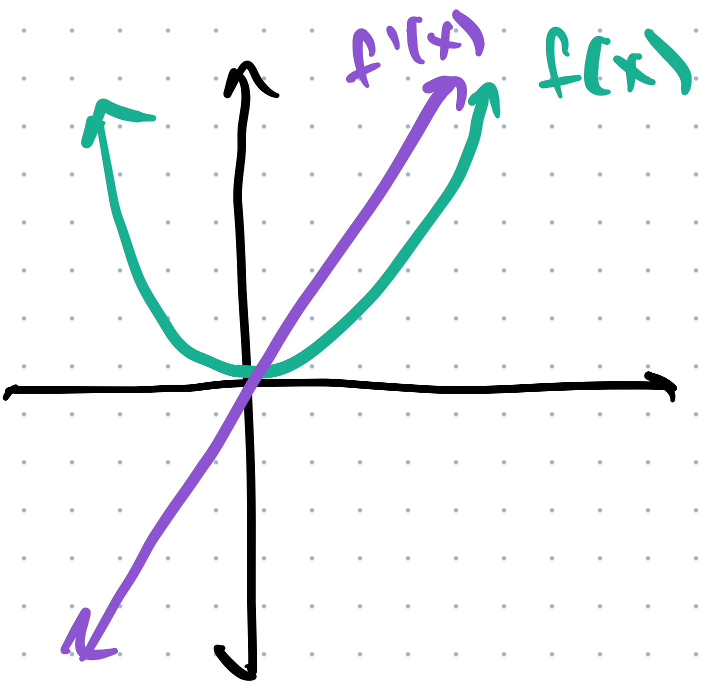


<u>Ex:</u> $f(x) = 2x - 1$

$f'(x) = 2$

$f'(x) = m$

Ex: If $f(x)=x^2$, what is $f'(x)$? $f'(x)=2x$



Things we know:

$f'(x)>0$ when $x>0$

$f'(x)=0$ when $x=0$

$f'(x)<0$ when $x<0$

Limit Definition: $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{(\cancel{x^2}+2xh+h^2)-\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}\frac{\cancel{h}\cdot(2x+h)}{\cancel{h}}=\lim_{h\to 0}(2x+h)$$

$h\cdot(2x+h)=2xh+h^2$

$$=2x$$

__Ex:__ Let $f(x) = \dfrac{1}{x-3}$. What is $f'(x)$?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$$

common denominator

$$\frac{1}{x+h-3} - \frac{1}{x-3} = \frac{1}{x+h-3}\left(\frac{x-3}{x-3}\right) - \frac{1}{x-3}\left(\frac{x+h-3}{x+h-3}\right)$$

$$= \frac{x-3}{(x+h-3)(x-3)} - \frac{x+h-3}{(x+h-3)(x-3)}$$

$$= \frac{(x-3)-(x+h-3)}{(x+h-3)(x-3)} = \frac{\cancel{x}\cancel{-3}\cancel{-x}-h\cancel{+3}}{(x+h-3)(x-3)} = \frac{-h}{(x+h-3)\cdot(x-3)}$$

Ex: Let $f(x) = \dfrac{1}{x-3}$. What is $f'(x)$?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{-h}{(x+h-3)\cdot(x-3)}\right)}{h}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\cdot(x+h-3)(x-3)} = \lim_{h\to 0} -\frac{1}{(x+h-3)(x-3)}$$

plug in $h=0$

$$-\frac{1}{(x+0-3)(x-3)} = -\frac{1}{(x-3)(x-3)} = \boxed{-\frac{1}{(x-3)^2}}$$

$f'(x)$

Sneak Peek of rules we'll see in Chapter 3

Power Rule

$f(x) = x^2 \rightsquigarrow f'(x) = 2x$

$f(x) = x^3 \rightsquigarrow f'(x) = 3x^2$

$f(x) = x^n \rightsquigarrow f'(x) = n \cdot x^{n-1}$

End of 2.3