Math 1450 - Calculus 1

Mon, Sept. 22

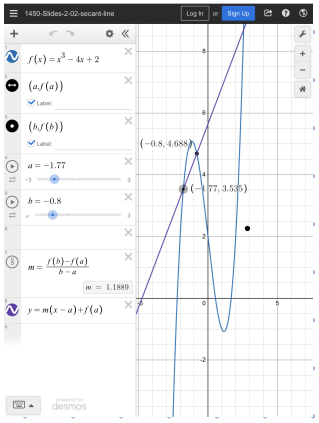
Announcements:

* Exam grades are in, we'll send out solutions and you are more than welcome to ask about the exam in office hours!
* Quiz 3 on Thursday / covers sugg. HW from 2.1, 2.2, 2.3
* Homework 4 due Thurs, covers 2.1 and 2.2

Today:

→ 2.2: The Derivative at a Point
→ 2.3: The Derivative Function

Office Hours
Mondays, 12-1 Cudahy 307
Wednesdays, 2-3
+ Help Desk! 12-1


1450-Slides-2-02-secant-line
Log In
or
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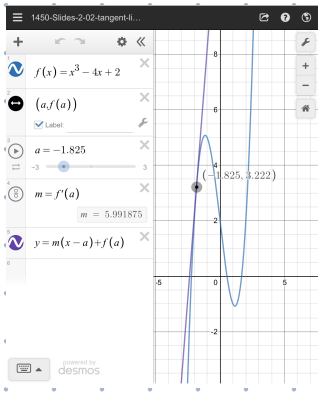
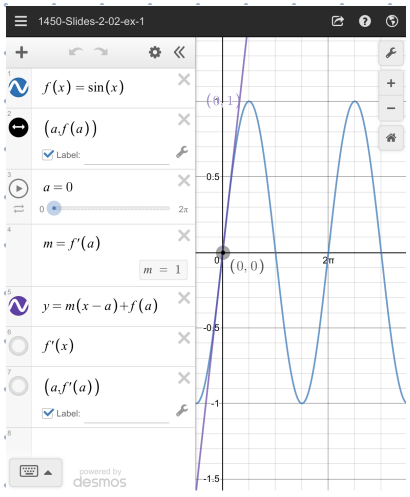
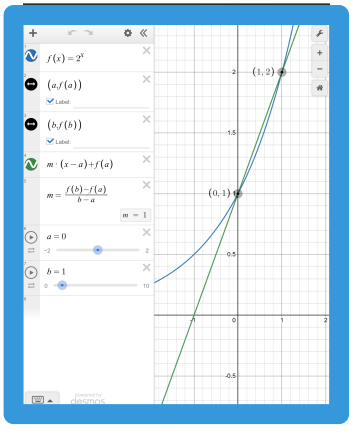
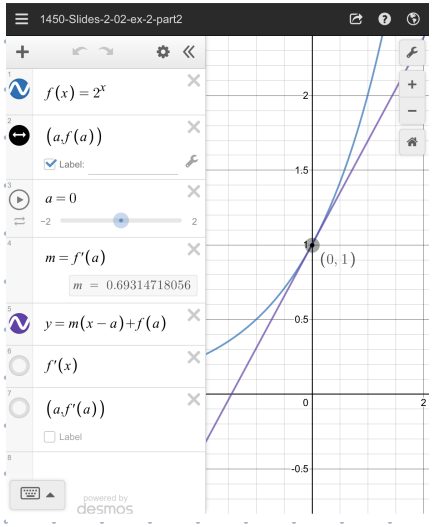
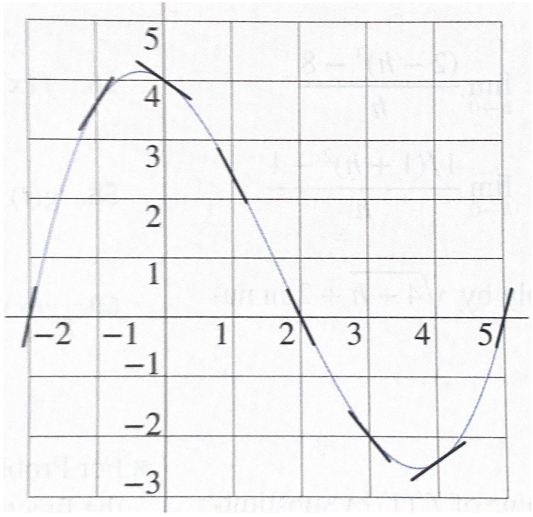
$f(x)=x^3-4x+2$
$(a,f(a))$
Label:
$(b,f(b))$
Label:
$a=-1.77$
$b=-0.8$
$m=\frac{f(b)-f(a)}{b-a}$
$m=1.1889$
$y=m(x-a)+f(a)$
$(-0.8,4.688)$
$(-1.77,3.535)$
powered by desmos

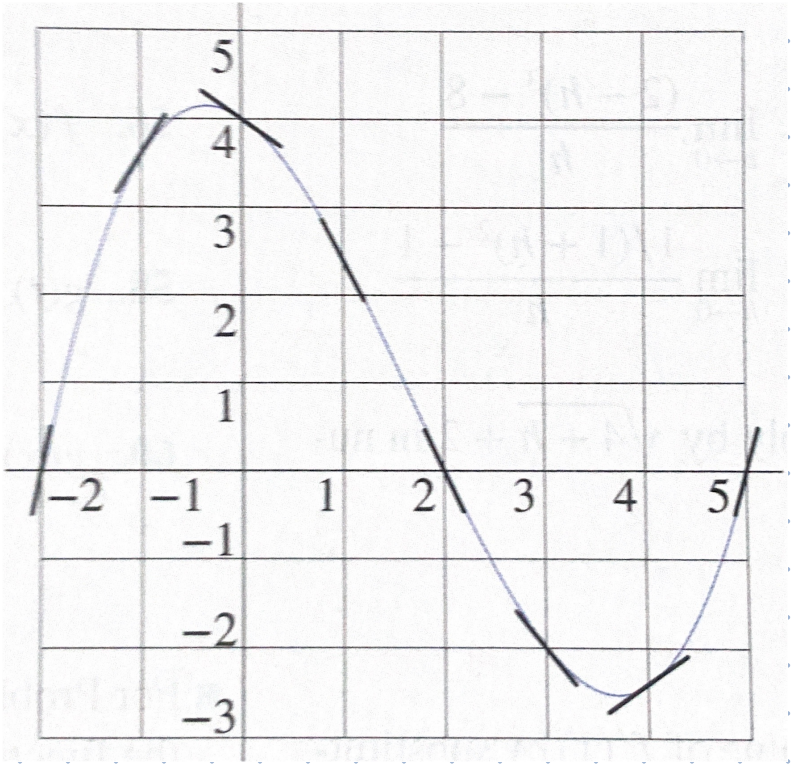

1450-Slides-2-02-tangent-li...
$f(x)=x^3-4x+2$
$(a,f(a))$
Label:
$a=-1.825$
$m=f'(a)$
$m=5.991875$
$y=m(x-a)+f(a)$
$(-1.825,3.222)$
powered by desmos


1450-Slides-2-02-ex-1
$f(x)=\sin(x)$
$(a,f(a))$
Label:
$a=0$
$m=f'(a)$
$m=1$
$y=m(x-a)+f(a)$
$f'(x)$
$(a,f'(a))$
Label:
$(0,0)$
$(\pi,1)$
powered by desmos

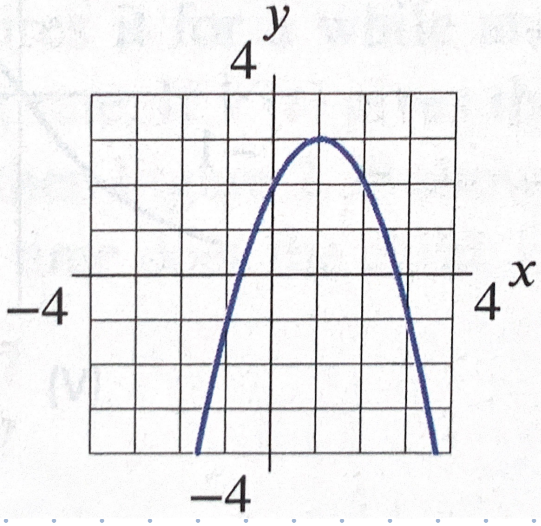

$f(x)=2^x$
$(a,f(a))$
$(b,f(b))$
$m\cdot(x-a)+f(a)$
$m=\frac{f(b)-f(a)}{b-a}$
$m=1$
$a=0$
$b=1$
$(1,2)$
$(0,1)$

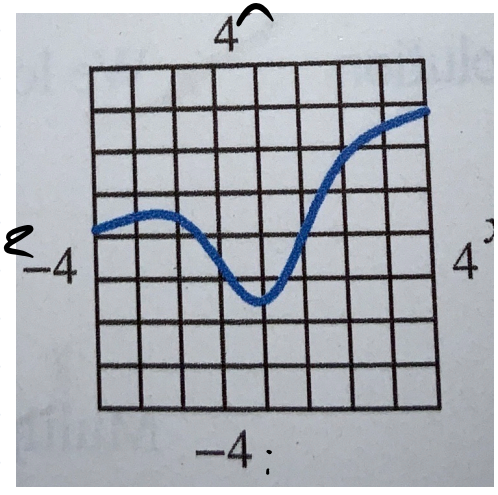

1450-Slides-2-02-ex-2-part2
$f(x)=2^x$
$(a,f(a))$
Label:
$a=0$
$m=f'(a)$
$m=0.69314718056$
$y=m(x-a)+f(a)$
$f'(x)$
$(a,f'(a))$
Label
$(0,1)$
powered by desmos





5.





Summary:

[derivative of $f(x)$ at $x=a$]

= [instantaneous RoC of $f(x)$ at $x=a$]

= $f'(a)$ "f-prime of a"

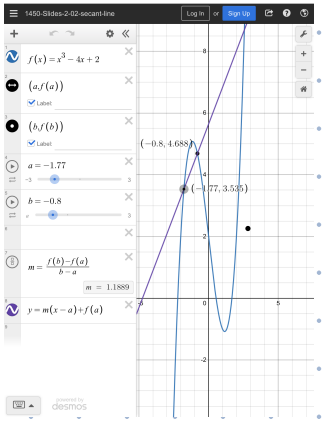
= $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Slopes

If you draw a line between two points on a function $(a, f(a))$ and $(b, f(b))$, then the slope of that line is:

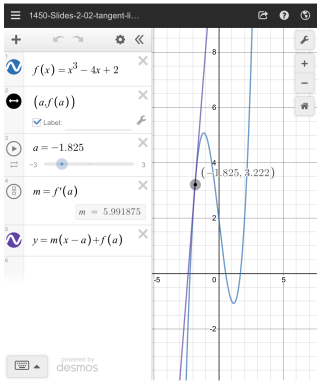
$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$ = average RoC of $f(x)$ from $t=a$ to $t=b$

"secant line"



The quantity $f'(a)$ is the slope of the curve at $x=a$.

The tangent line of $f(x)$ at the point $x=a$ is the line with slope $f'(a)$ that passes through the point $(a, f(a))$



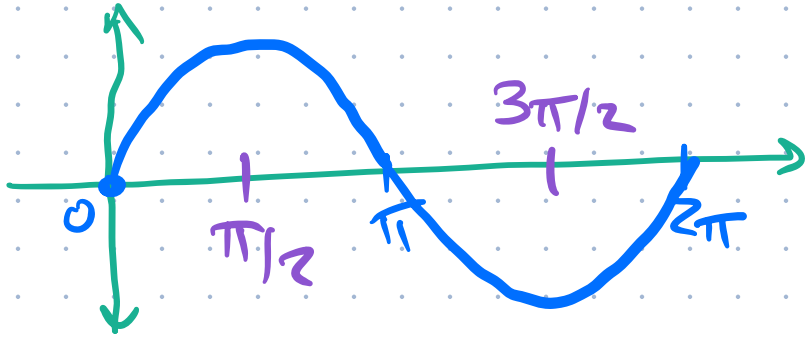
$f'(a) > 0$ means the function is increasing "going up" at the x-value a

$f'(a) < 0 \Rightarrow$ decreasing at $x = a$

$f'(a) = 0 \Rightarrow$ graph is flat at $x = a$

Ex: Let $f(x) = \sin(x)$.

Is $f'(\pi)$ positive, negative, or 0?



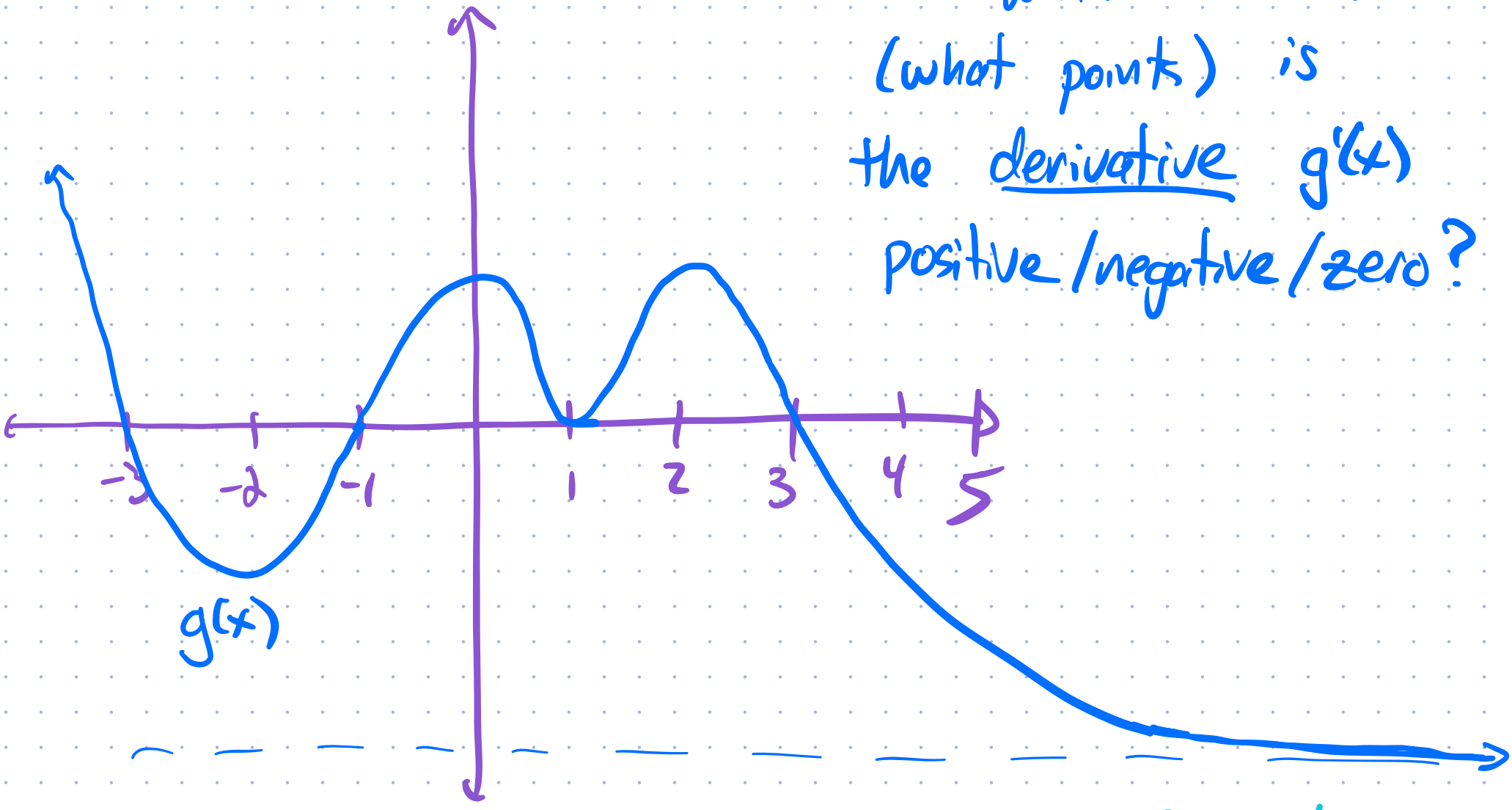
$f(x)$

$f(\pi) = 0$

$f'(\pi)$ is negative

Group Work

On what intervals (what points) is the derivative $g'(x)$ positive / negative / zero?

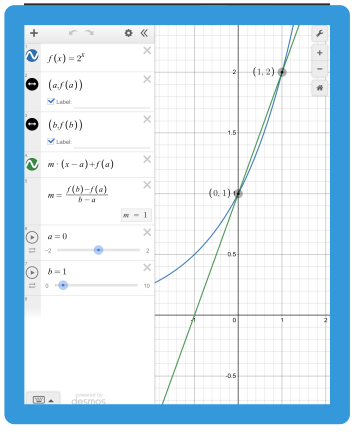


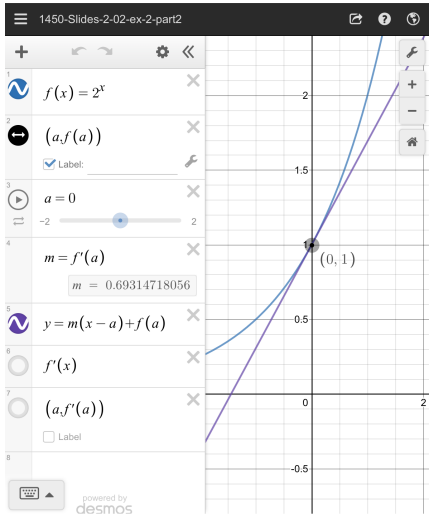
positive: $(-2, 0)$, $(1, 2)$

negative: $(-\infty, -2)$, $(0, 1)$, $(2, \infty)$

zero: at -2, at 0, at 1, at 2

Ex: Estimate the derivative of $f(x) = 2^x$ at $x = 0$ with a graph.





$\approx 0.693\ldots$

$\ln(2)$

Part 2: Find the equation for the tangent line of $f(x) = 2^x$ at $x = 0$.

$f'(0) \approx 0.693$

Formula for the line with slope m that passes through the point (c, d):

$$y = m \cdot (x - c) + d$$

$(0, f(0)) = (0, 1)$

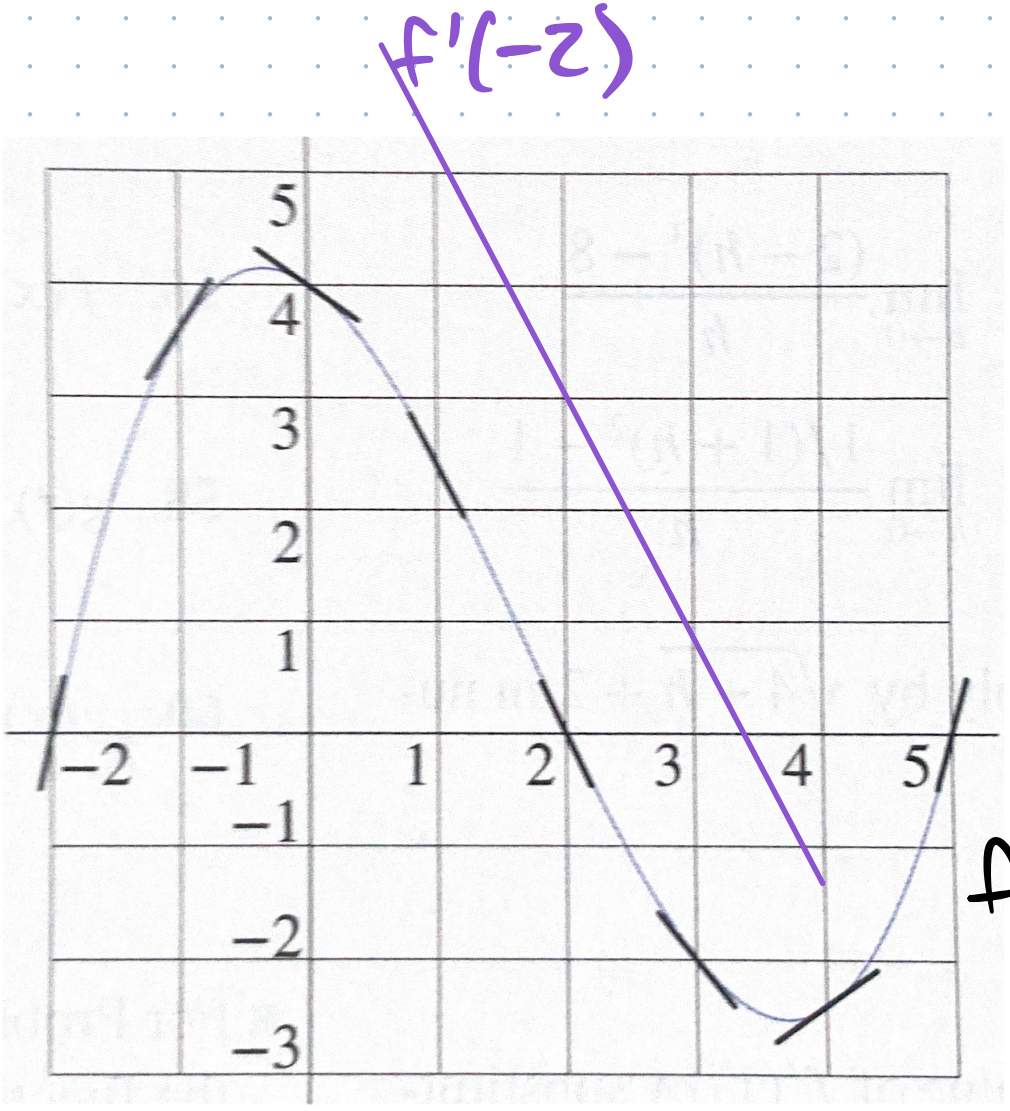
$$y = 0.693 \cdot (x - 0) + 1$$

$$= 0.693x + 1$$ — equation of the T.L. at $x = 0$

Section 2.3 – The derivative function

(Same idea as Section 2.2, in more detail)

Ex: Estimate $f'(x)$ at $x = -2, -1, 0, 1, 2, 3, 4, 5$.



x	f(x)	f'(x)
-2	0	5
-1	3.5	2
0	4	-0.8
1	2.5	-2.5
2	0	-2.5
3	-2	-1
4	-2.5	0.8
5	0	5