Math 1450 - Calculus 1

Fri, Sept. 19

Announcements:

* Exam grading in progress!
* Quiz 3 next Thursday, Sept. 25
* Homework 4 due next Thurs, covers 2.1 and 2.2

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 2.1: How do we measure speed?

→ 2.2: The Derivative at a Point

Another Example: Throwing a Grapefruit

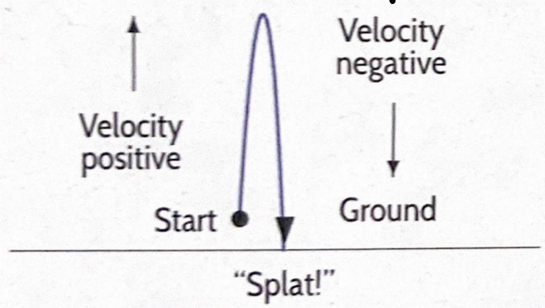


Table 2.1 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

Avg ~~speed~~ velocity over 1st second:

$$\frac{\text{dist. traveled}}{\text{time}} = \frac{90-6 \text{ feet}}{1 \text{ second}} = 84 \frac{\text{ft.}}{\text{sec}}$$

Avg ~~speed~~ velocity over 2nd second: $t=1 \to t=2$

$$\frac{142-90 \text{ ft}}{1 \text{ sec}} = 52 \frac{\text{ft}}{\text{sec}}$$

"velocity" has a negative sign for going down"
"speed" is always positive

Speed vs. Velocity:

Velocity incorporates direction

Speed ignores direction

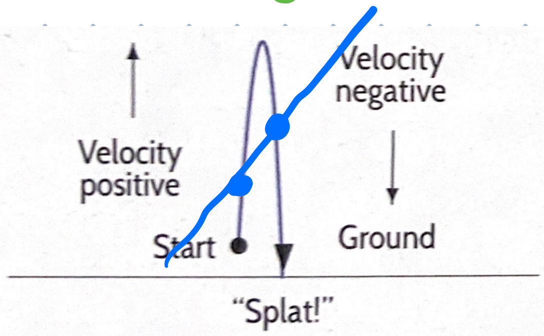
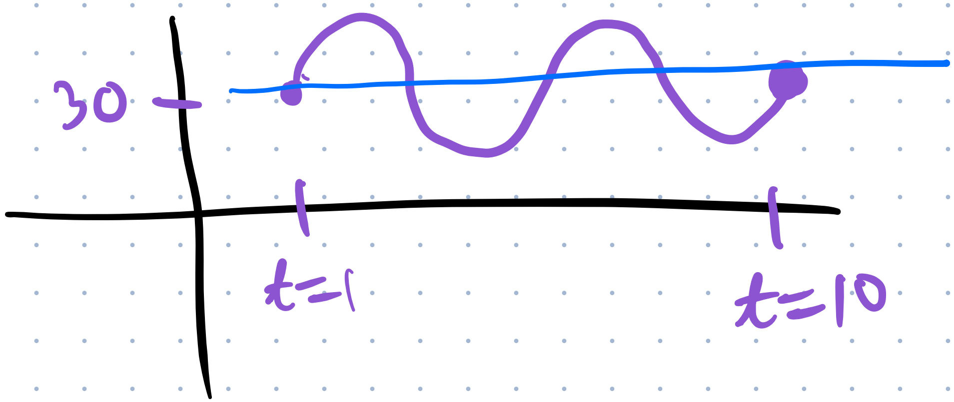


Table 2.1 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

Avg velocity from 4 sec → 5 sec

$$\frac{106-150}{5-4} = -44 \ \frac{ft}{sec}$$



average velocity:

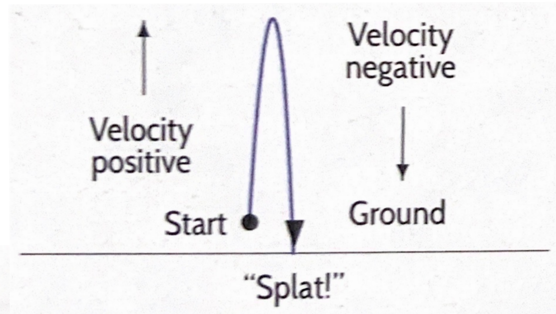
$$\frac{30-30}{10-1} = 0 \ \frac{ft}{se}$$

* Suppose $s(t)$ is a function that tells you the position of an object at time t.

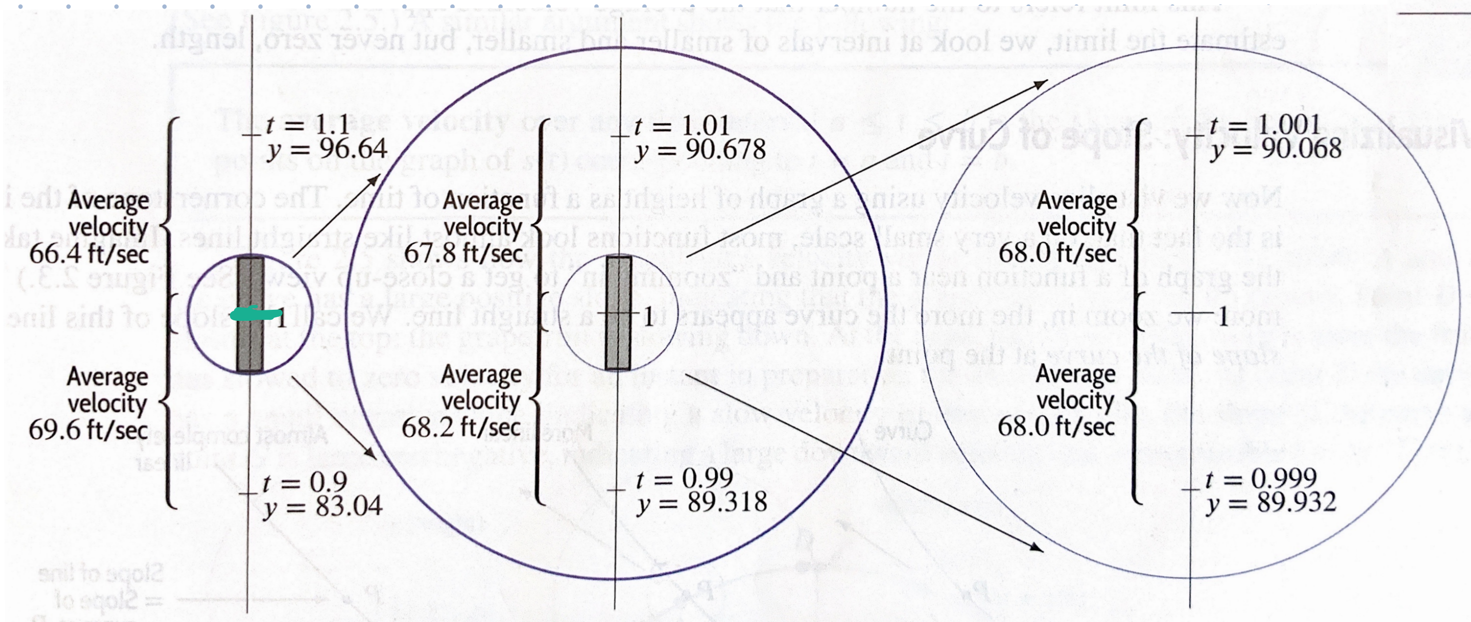
Average velocity from $t=a$ to $t=b$ is:

$$\text{Avg vel} = \frac{\text{change in pos}}{\text{change in time}} = \frac{\overset{\text{end position}}{s(b)} - \overset{\text{start position}}{s(a)}}{\underset{\text{end time}}{b} - \underset{\text{start time}}{a}}$$

Let's estimate the instantaneous velocity of the grapefruit at $t=1$ with $s(t) = 6 + 100t - 16t^2$



$s(1) = 6 + 100(1) - 16(1)^2 = 90$



Instantaneous velocity at $t=1$ seems like $68 \frac{ft}{sec}$

Feels kind of like we're doing a limit!

To estimate the instantaneous velocity at $t=a$

$h=1$: $\dfrac{s(a+1)-s(a)}{(a+1)-a} = \dfrac{s(a+1)-s(a)}{1}$ avg. over 1 second

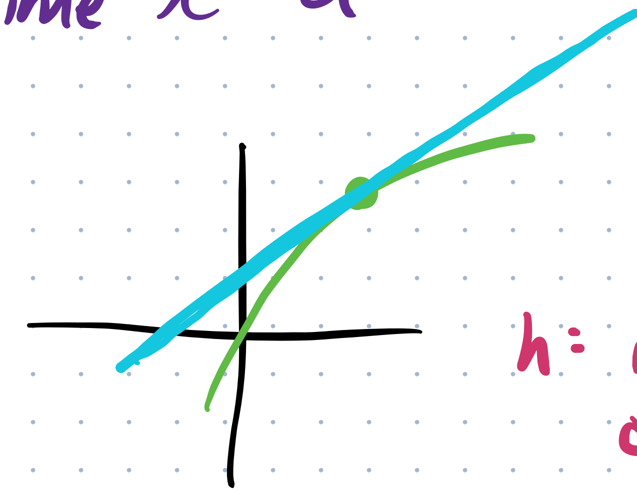
$h=0.1$: $\dfrac{s(a+0.1)-s(a)}{(a+0.1)-a} = \dfrac{s(a+0.1)-s(a)}{0.1}$ avg over 0.1 sec.

$h=0.00001$ $\dfrac{s(a+0.00001)-s(a)}{0.00001}$

Instantaneous Velocity at time $t=a$

=

$$\lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$$



h: interval of time

This is a "derivative", one of the two fundamental tools of Calculus (more in 2.3)

This is really like computing the slope of the curve at a point (more in 2.2)

(2.1 has some discussion - skipping for now)

Example:

Suppose the position of a car in feet after t seconds is given by $s(t) = t^2$. What is the instantaneous velocity of the car after 3 seconds?

$$\lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

$$= \lim_{h \to 0} \frac{s(3+h) - s(3)}{h}$$

$$= \lim_{h \to 0} \frac{(3+h)^2 - (3)^2}{h}$$

$$= \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = 6$$

$$\boxed{6 \text{ ft/sec}}$$

Another example if time:

11. In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s = \sin(2t)$.

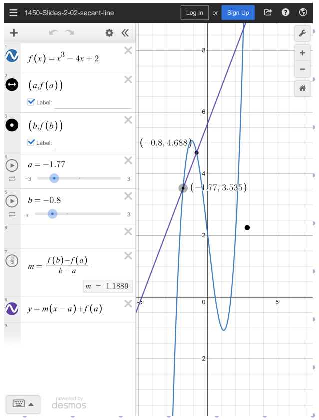
(a) Find the average velocity between $t = 1$ and $t = 1 + h$ if:

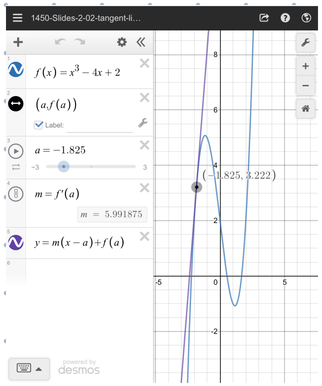
(i) $h = 0.1$, (ii) $h = 0.01$, (iii) $h = 0.001$.

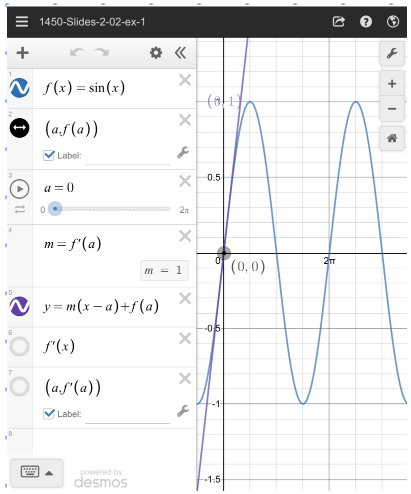
(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

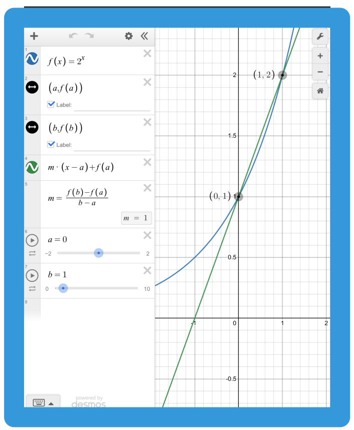
Suggested Homework:

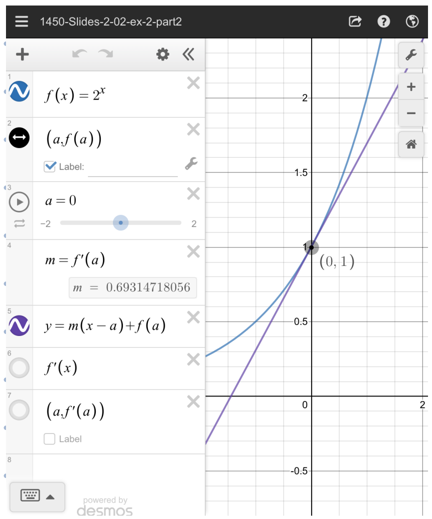
1, 2, 3, 4, 5, 11, 13, 14, 15, 16, 23, 31


1450-Slides-2-02-secant-line
Log In
Sign Up
$f(x)=x^3-4x+2$
$(a,f(a))$
$(b,f(b))$
$a=-1.77$
$b=-0.8$
$m=\frac{f(b)-f(a)}{b-a}$
$m=1.1889$
$y=m(x-a)+f(a)$
$(-0.8, 4.688)$
$(-1.77, 3.535)$


1450-Slides-2-02-tangent-li...
$f(x)=x^3-4x+2$
$(a,f(a))$
$a=-1.825$
$m=f'(a)$
$m=5.991875$
$y=m(x-a)+f(a)$
$(-1.825, 3.222)$


1450-Slides-2-02-ex-1
$f(x)=\sin(x)$
$(a,f(a))$
$a=0$
$m=f'(a)$
$m=1$
$y=m(x-a)+f(a)$
$f'(x)$
$(a,f'(a))$
$(0,1)$
$(0,0)$
2π


$f(x)=2^x$
$(a,f(a))$
$(b,f(b))$
$m\cdot(x-a)+f(a)$
$m=\frac{f(b)-f(a)}{b-a}$
$m=1$
$a=0$
$b=1$
$(1,2)$
$(0,1)$


1450-Slides-2-02-ex-2-part2
$f(x)=2^x$
$(a,f(a))$
$a=0$
$m=f'(a)$
$m=0.69314718056$
$y=m(x-a)+f(a)$
$f'(x)$
$(a,f'(a))$
$(0,1)$

Section 2.2 - The derivative at a point

Average rate of change (velocity) from time a to time b.

$$\frac{\Delta \text{position}}{\Delta \text{time}} = \frac{f(b)-f(a)}{b-a}$$

Δ = "change in"

Average RoC over a small time period $t=a$ to $t=a+h$

h is a small #

$$\frac{\Delta \text{position}}{\Delta \text{time}} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous RoC

$$\lim_{h\to 0}\left(\text{avg. RoC from } t=a \text{ to } t=a+h\right) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is called "the derivative of the function $f(t)$ at the point $t=a$".

Summary:

[derivative of $f(x)$ at $x=a$]

= [instantaneous RoC of $f(x)$ at $x=a$]

$= f'(a)$ "f-prime of a"

$$= \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

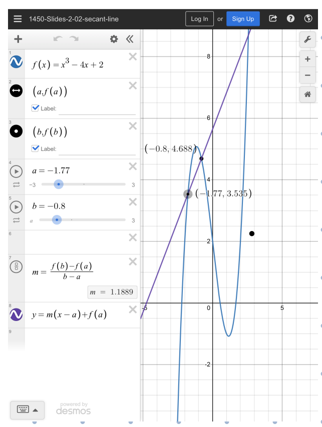
Slopes

If you draw a line between two points on a function $(a, f(a))$ and $(b, f(b))$, then the slope of that line is:

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} = \text{average RoC of } f(x)$$

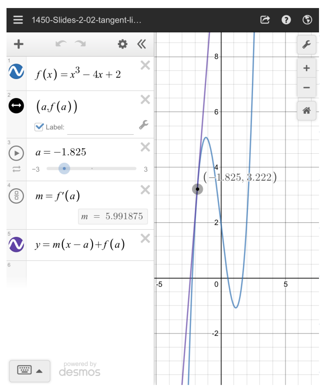
from $t=a$ to $t=b$

"secant line"



The quantity $f'(a)$ is the <u>slope of the curve</u> at $x=a$.

The <u>tangent line</u> of $f(x)$ at the point $x=a$ is the line with slope $f'(a)$ that passes through the point $(a, f(a))$



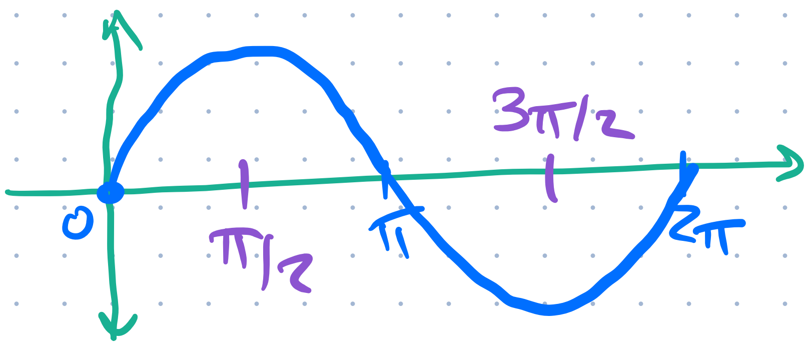
$f'(a) > 0$ means the function is increasing "going up" at the x-value a

$f'(a) < 0 \Rightarrow$ decreasing at $x = a$

$f'(a) = 0 \Rightarrow$ graph is flat at $x = a$

Ex: Let $f(x) = \sin(x)$.

Is $f'(\pi)$ positive, negative, or 0?



$f(x)$

$f(\pi) = 0$

$f'(\pi)$ is negative