Math 1450 - Calculus 1

Wed, Sept. 17

Announcements:

* Exam 1 - tonight, 5pm-6pm, this room
  * study guide on course website!
  * covers 1.1-1.9
  * calculators allowed (nothing with wifi/bluetooth)
* Activity in discussion on Thursday

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!
12-1

Today:

→ 2.1: How do we measure speed?

→ Review

$$\lim_{x\to\infty} \frac{x^2+4}{x-3} = \infty$$

$$= \lim_{x\to\infty} \frac{x^2}{x}$$

$$= \lim_{x\to\infty} x$$

$$= \infty$$

$$\lim_{x\to-\infty} \frac{x^2+4}{x-3} \quad \begin{matrix} +\infty \\ -\infty \end{matrix}$$

$$\lim_{x\to\infty} \frac{5x^2+4}{3x^2-100x} = \frac{5}{3}$$

$$\lim_{x\to\infty} \frac{5\cdot\cancel{x^2}\ \cancel{+4}}{3\cdot\cancel{x^2}\ \cancel{-100x}} = \frac{5}{3}$$

$$\lim_{x\to\infty} \frac{5\cdot 2^x+4}{3\cdot 2^x-100,}$$

$$= \lim_{x\to\infty} \frac{5\cdot 2^x}{3\cdot 2^x}$$

$$= \lim_{x\to\infty} \frac{5}{3}$$

$$= \frac{5}{3}$$

$$\lim_{x\to\infty} \frac{5\cdot 2^x+4}{3\cdot 3^x-100x}$$

$$= \lim_{x\to\infty} \frac{5\cdot 2^x}{3\cdot 3^x}$$

$$= \lim_{x\to\infty} \frac{5}{3}\cdot\left(\frac{2}{3}\right)^x$$

$$= 0$$

$$f(x) = \begin{cases} \frac{3}{4}x^2 & \text{if } x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$$

Find a value of k that makes $f(x)$ continuous.

At $x=2$, one part is $k \cdot 2^2 = 4k$

other part is 3

$$4k = 3 \Rightarrow k = \frac{3}{4}$$

Simplify $\log_3(9^7 \cdot x)$

$= \log_3(9^7) + \log_3(x)$

$= 7 \cdot \log_3(9) + \log_3(x)$

$= 7 \cdot \log_3(3^2) + \log_3(x)$

$= 7 \cdot (2 \cdot \overset{1}{\cancel{\log_3(3)}}) + \log_3(x)$

$= 7 \cdot 2 + \log_3(x)$

$= 14 + \log_3(x)$

$\ln = \log_e$ $\qquad e \approx 2.71\ldots$

$$\ln(9^7 \cdot x)$$

$$= \ln(9^7) + \ln(x)$$

$$= 7 \cdot \ln(9) + \ln(x)$$

$$= 7 \cdot \ln(3^2) + \ln(x)$$

$$= 14 \ln(3) + \ln(x)$$

Review Q16

pop 40,000

5 hours ↘ 35000

↘ exp

~~↘ linearly~~

pop = 20,000

$P(x) = 40{,}000 \cdot a^x$ what is a?

$35000 = 40000 \cdot a^5$

$\frac{35}{40} = \frac{7}{8} = a^5 \Rightarrow a = \sqrt[5]{7/8} = \left(\frac{7}{8}\right)^{1/5}$

≈ 0.974

$$20000 = 40000 \cdot \left(\left(\frac{7}{8}\right)^{1/5}\right)^x$$ solve for x

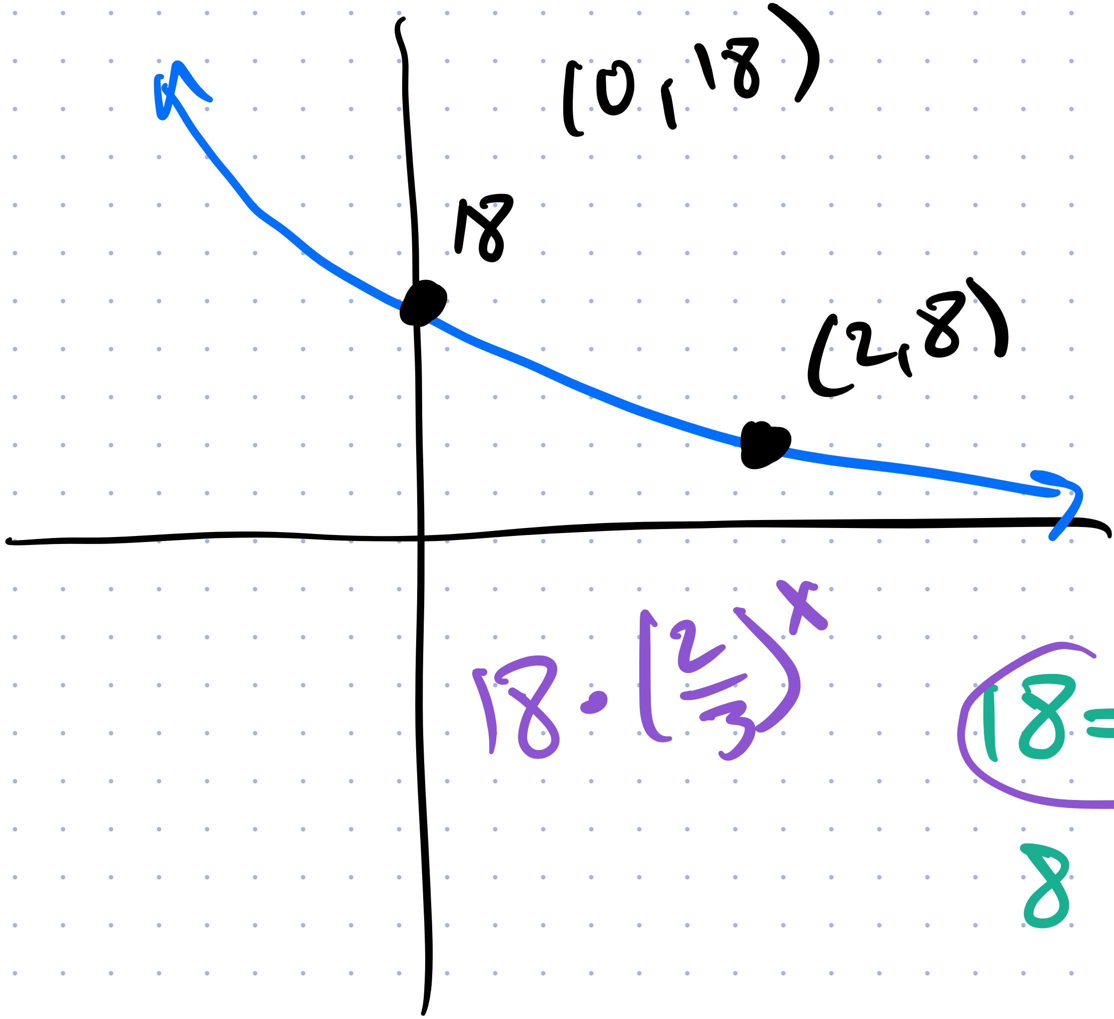
$$\frac{1}{2} = \left(\left(\frac{7}{8}\right)^{1/5}\right)^x$$

$$\log_{\left(\frac{7}{8}\right)^{1/5}}\left(\frac{1}{2}\right) = x$$

$$\frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\left(\frac{7}{8}\right)^{1/5}\right)} \approx 25 \text{ hours}$$

Exponential $P_0 \cdot a^x$

"Continuous" $P_0 \cdot e^{k \cdot x}$



$$P(x) = P_0 \cdot a^x$$

$$P(0) = 18$$

$$P(2) = 8$$

$$18 \cdot \left(\frac{2}{3}\right)^x$$

$$18 = P_0 \cdot \cancel{a^0} \; 1$$

$$8 = P_0 \cdot a^2$$

$$8 = 18 \cdot a^2$$

$$\frac{8}{18} = \frac{4}{9} = a^2 \Rightarrow a = \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Section 2.1 - How Do We Measure Speed?

Average Speed is an easy concept.

If you ran 12 miles in 1.5 hours, then your average speed was $\frac{12 \text{ mi}}{1.5 \text{ hr}} = 8 \frac{\text{mi}}{\text{hr}}$.

Instantaneous Speed is weirder!

How fast were you going at exactly 3 minutes and 47 seconds?

(How do speedometers do it?)

If you had an extremely precise GPS / stopwatch combo, how could you <u>estimate</u> your instantaneous speed at 3 min, 47 sec? (= 227 seconds)

Calculate <u>average</u> speed over smaller and smaller intervals around 227 seconds.

Average speed from 217 seconds → 237 seconds:

$$\frac{\text{distance traveled in those 20 seconds}}{\text{20 seconds}}$$

More precise:

Average speed from 226 → 228 seconds:

$$\frac{\text{distance traveled in those 2 seconds}}{\text{2 seconds}}$$

Average from 226.99 → 227.01 seconds

$$\frac{\text{dist. traveled in those 0.02 seconds}}{\text{0.02 seconds}}$$

Another Example: Throwing a Grapefruit

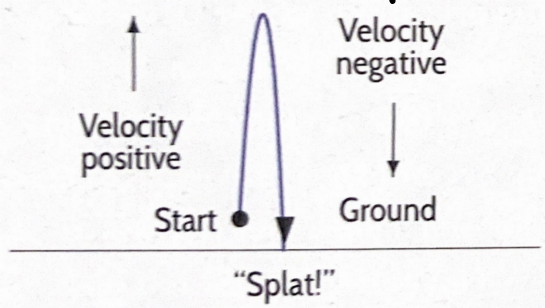


Table 2.1 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

Avg ~~speed~~ velocity over 1st second:

$$\frac{\text{dist. traveled}}{\text{time}} = \frac{90-6 \text{ feet}}{1 \text{ second}} = 84 \frac{\text{ft.}}{\text{sec}}$$

Avg ~~speed~~ velocity over 2nd second: $t=1 \to t=2$

$$\frac{142-90 \text{ ft}}{1 \text{ sec}} = 52 \frac{\text{ft}}{\text{sec}}$$

"Velocity" has a negative sign for going down"
"speed" is always positive

Speed vs. Velocity:

Velocity incorporates direction

Speed ignores direction

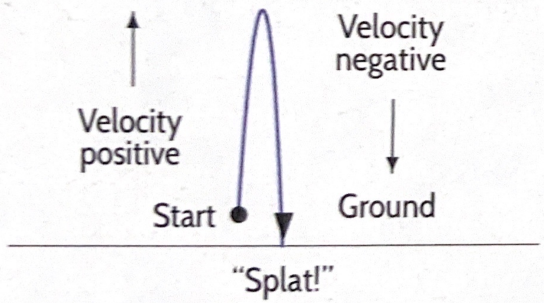


Table 2.1 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

Avg velocity from 4 sec → 5 sec

$$\frac{106-150}{5-4} = -44 \ \frac{ft}{sec}$$