Math 1450 - Calculus 1

Mon, Sept. 15

Announcements:

* HW 3 due ~~Tuesday~~ tomorrow, 11:59pm
  covers 1.7 - 1.9

* Exam 1 - Wednesday, 5pm-6pm, this room
  * study guide on course website!
  * covers 1.1-1.9
  * calculators allowed (nothing with wifi/bluetooth)

* Activity in discussion on Thursday

Today:

→ 1.9: Further Limit Calculations Using Algebra
→ Review, if time

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Section 1.9 – Further Limit Calculations Using Algebra

We'll focus in this section on limits of functions of the form $\frac{f(x)}{g(x)}$ at $y = \pm\infty$ and

potentially horizontal asymptotes

at points where $g(x) = 0$.

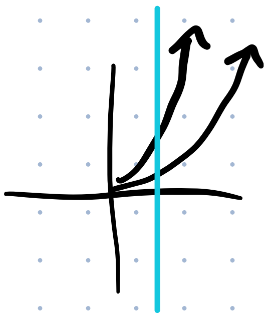
potentially vertical asymptotes

(1) $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ and $\lim_{x \to -\infty} \frac{f(x)}{g(x)}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

If $g(x)$ "grows faster" than $f(x)$ as $x \to \infty$ or $-\infty$, then the limits are 0.

Examples



Which grows faster as $x \to \infty$?

x^6 vs. x^3

$-x^6$ vs x^3 (ignore + and − signs)

x^3+2x+1 vs $1+x^2+x^4$

$x^{1/2} = \sqrt{x}$ vs x

As $x \to \infty$

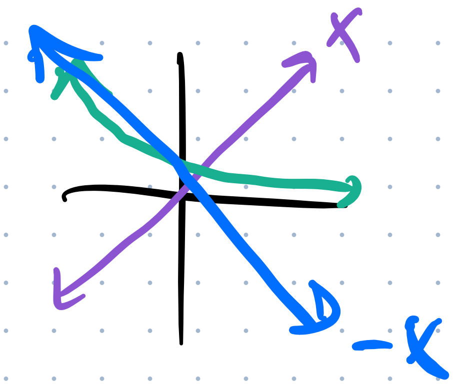
exp. growth

polynomial

$(1.1)^x$ vs x^{1000}

(eventually)

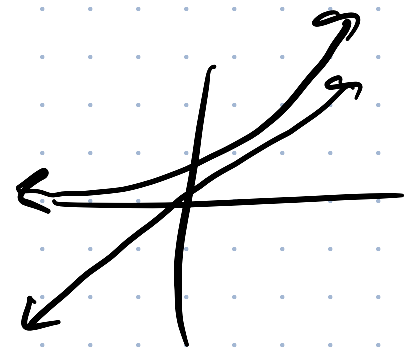
$\left(\frac{1}{2}\right)^x$ vs. x



As $x \to -\infty$

$\left(\frac{1}{2}\right)^x$ vs. x

x^6 vs. x^3

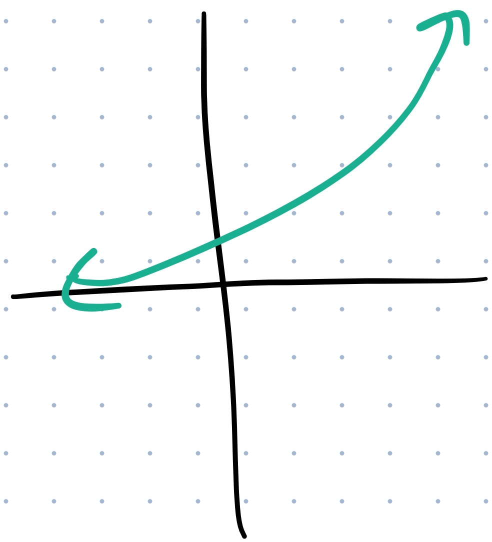


lim as $x \to -\infty$

$x^3 + 2x + 1$ vs. $1 + x^2 + x^4$

$x^{1/2}$ — $\cancel{\sqrt{x} \text{ vs. } x}$

not defined



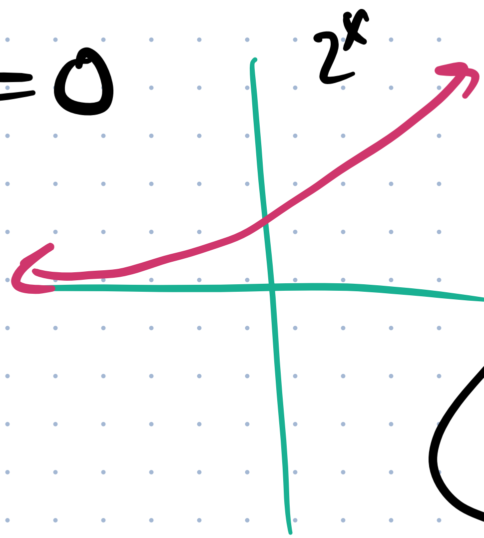
3^x vs. x^{100}

Examples:

$$\lim_{x\to\infty} \frac{100x^3+3x+5}{0.01x^6+1} = 0$$

because the bottom "grows faster than the top" as $x\to\infty$

$$\lim_{x\to-\infty} \frac{100x^3+3x+5}{0.01x^6+1} = 0$$



$$\lim_{x\to\infty} \frac{x^{100}+x^{10}}{2^x+1} = 0$$

$$\lim_{x\to-\infty} \frac{x^{100}+x^{10}}{2^x+1} = \infty \quad \left(\frac{\text{pos big \#}}{\text{pos small \#}}\right)$$

$$2^{-100}+1 = \frac{1}{2^{100}}+1$$

$$\lim_{x\to-\infty} \frac{x^{100}+x^{10}}{2^x-1} = -\infty \quad \left(\frac{\text{pos big}}{\text{neg small}}\right)$$

(2) If f(x) and g(x) grow "equally fast" then you eliminate the slower terms and look at what's left.

$$\lim_{x \to \infty} \frac{5x^2 \cancel{-3x+1}}{2x^2 \cancel{+17}} = \lim_{x \to \infty} \frac{5\cancel{x^2}}{2\cancel{x^2}} = \boxed{\frac{5}{2}}$$

$$\lim_{x \to -\infty} \frac{5x^2 \cancel{-3x+1}}{2x^2 \cancel{+17}} = \lim_{x \to -\infty} \frac{5\cancel{x^2}}{2\cancel{x^2}} = \boxed{\frac{5}{2}}$$

(3) Limits where the denominator is 0

$$\lim_{x \to c} \frac{f(x)}{g(x)} \quad \text{where} \quad g(c) = 0.$$

Ex: $\displaystyle\lim_{x \to 3} \frac{x+1}{x-3}$ What happens near 3?

Plug in a # a little bigger than 3

Numer: a little bigger than 4

Denom: a little bigger than 0

$$\frac{4}{\text{very small pos \#}} = \text{very big pos \#}$$

$$\lim_{x \to 3^+} \frac{x+1}{x-3} = +\infty$$

$$\frac{4 \text{ ish}}{(\text{very small neg \#})} = \text{very big neg \#}$$

$$\lim_{x \to 3^-} \frac{x+1}{x-3} = -\infty$$

General Rule for $\lim_{x \to c} \frac{f(x)}{g(x)}$

If (1) $f(x)$ is continuous at $x=c$

(2) $f(c) \neq 0$

(3) $g(c) = 0$

then the one-sided limits of $\frac{f(x)}{g(x)}$ as $x \to c$ are both $+\infty$ or $-\infty$.

To figure out which one, think about whether f and g are pos/neg to the left and right of c.

Squeeze Theorem

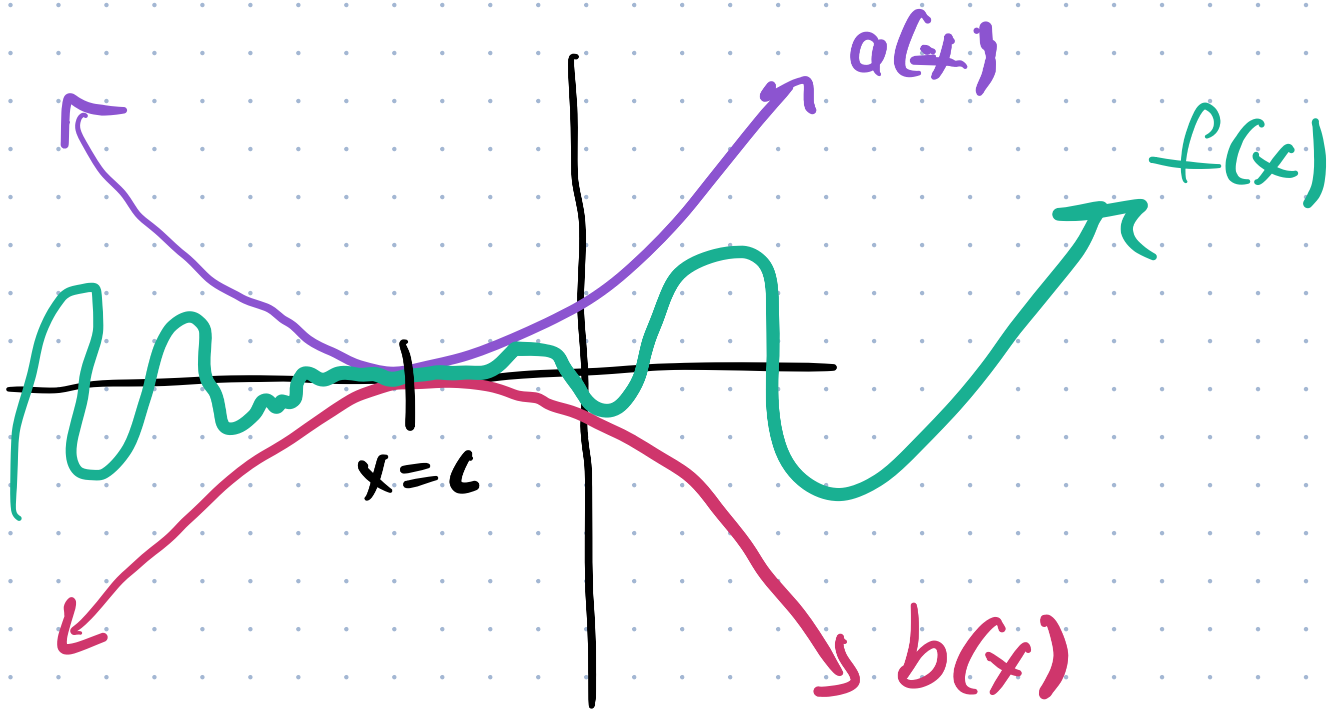
Suppose you have two functions, $a(x)$ and $b(x)$ such that $a(x) \geq b(x)$ for all x. Assume also that at $x=c$ we have

$$\underbrace{\lim_{x \to c} a(x) = \lim_{x \to c} b(x)}_{} = L.$$

$a(x)$ and $b(x)$ meet at value L as $x \to c$.

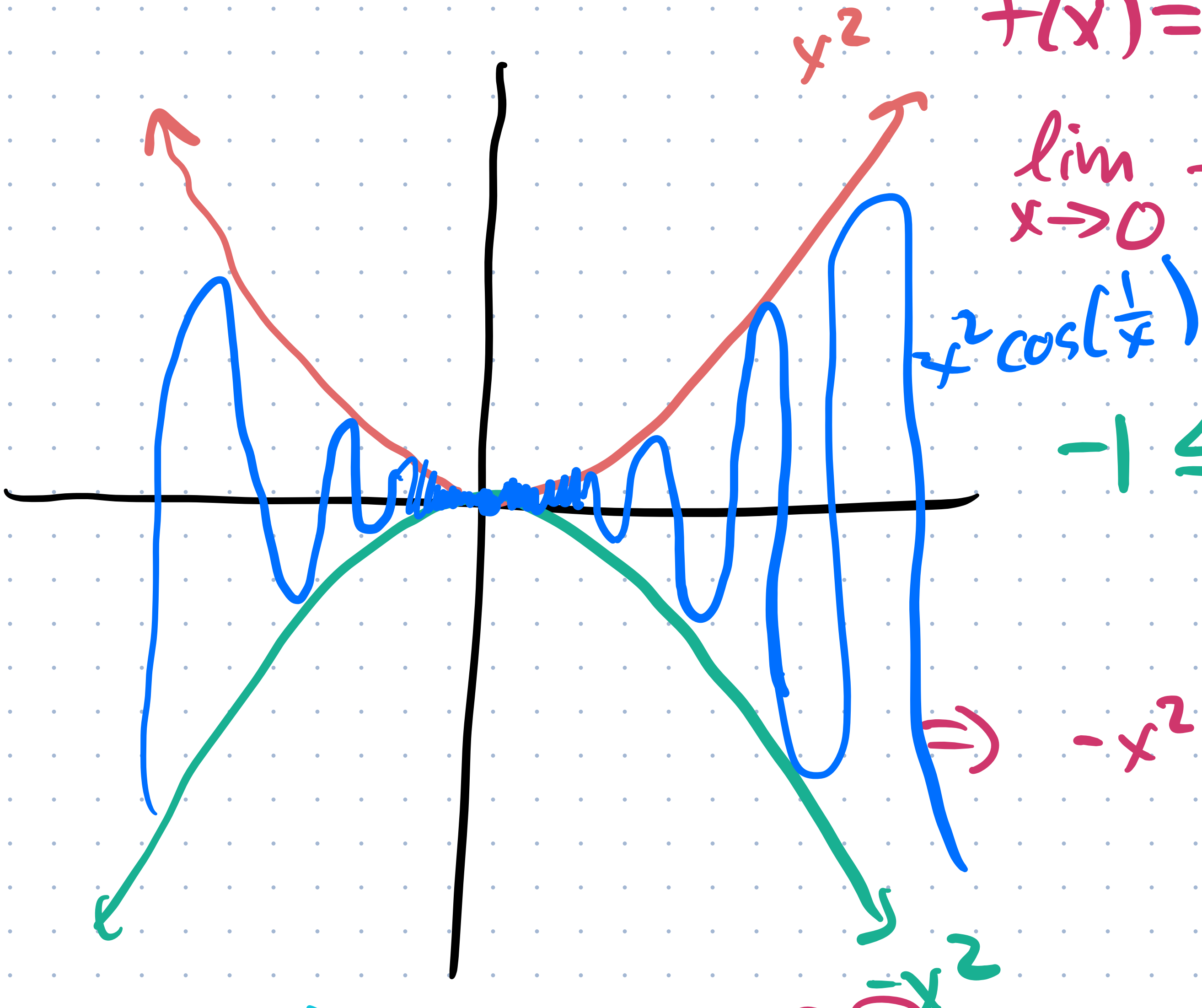
Then if $f(x)$ is some function that's always <u>between</u> $a(x)$ and $b(x)$, then

$$\lim_{x \to c} f(x) = L.$$



limits of a(x), b(x), f(x) at x = c
are all the same

f(x) gets squeezed between a and b.



$$f(x) = x^2 \cdot \cos\left(\frac{1}{x}\right)$$

$$\lim_{x\to 0} f(x) \; ? \;= 0$$

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$

for all x

$$\Rightarrow \; -x^2 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$$

$$\overset{=0}{\lim_{x\to 0}(-x^2)} \le \overset{=0}{\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right)} \le \overset{=0}{\lim_{x\to 0}(x^2)}$$