Math 1450 – Calculus 1

Wed, Aug 27

Announcements:

* No discussion tomorrow
* Calculators – Graphing calc allowed for exams/activities, but you really only need a scientific calculator.
  nothing with wifi capabilities
* First HW due Thurs, Sept 4 – Wiley Plus
* First quiz same day (no calculators for quizzes)
* Course website!
  jaypantone.com → Math 1450

Office Hours

Mondays, 12-1

Wednesdays, 2-3

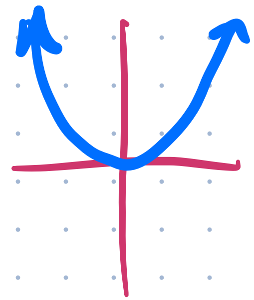
+ Help Desk!

Today:

→ 1.1: Functions and Change
→ 1.2: Exponential Functions

Section 1.1 - Functions and Change

In math before calculus, you learn about all kinds of functions



- lines
- parabolas
- exponentials
- logarithms
- etc

A function is just a predefined rule that transforms numbers into other numbers.

Think of a function as a <u>black box</u> that you feed input #s to, and it does something and it gives you back the output #s.

$$\underset{x}{\text{input \#}} \longrightarrow \boxed{\quad f \quad} \longrightarrow \underset{f(x)}{\text{output \#}}$$

The most important part is that you can only get one output for each input, not multiple.

If $3 \to \boxed{f} \to 5$, then $f(\overset{\text{input}}{3}) = \overset{\text{output}}{5}$ "f of 3 equals 5"
and $f(3)$ is <u>always</u> 5 for this function.

Book's Definition:

A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

Domain: Set of valid inputs
Range: Set of outputs you get

Domain: Set of valid inputs
Range: Set of outputs you get

The domain can be explicit (we specify the domain when we define the function) or implicit (all real #s that make sense when we plug them in).

A few notes before we do some examples:

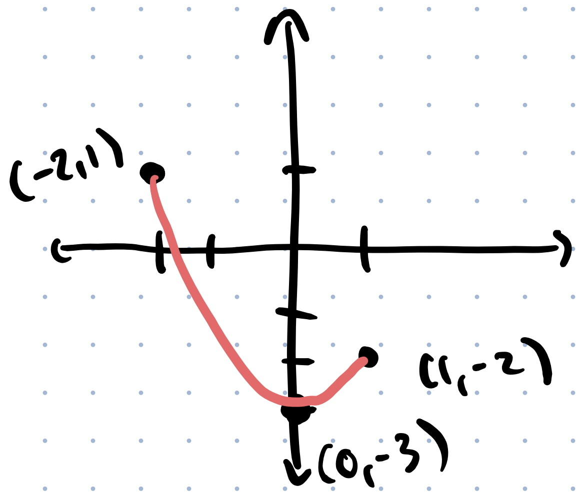
- "f" is just a common name for a function. We can use $g(x)$, $h(x)$, $B(x)$, $\alpha(x)$, ...
- It's common to write "y" in place of "$f(x)$", or sometimes "$y(x)$".

$y =$

Example:

explicit

$f(x) = x^2 - 3$, on domain $-2 \le x \le 1$



Domain: $-2 \le x \le 1$

Range: $-3 \le x \le 1$

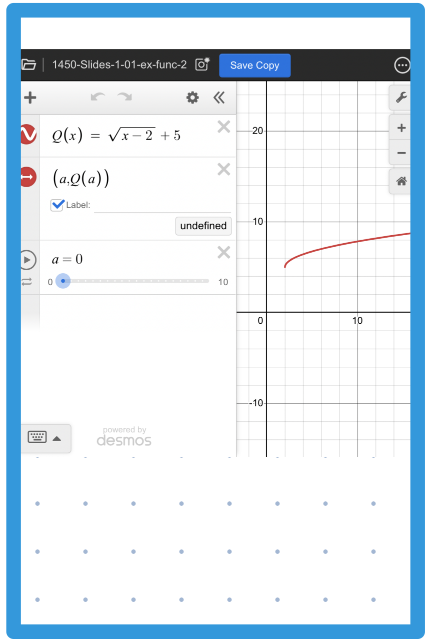
Example:

$Q(x) = \sqrt{x-2} + 5$ (implicit domain!)

$$Q(11) = \sqrt{11-2} + 5$$
$$= \sqrt{9} + 5$$
$$= 3+5$$
$$= \boxed{8}$$

where is $x-2 \geq 0$

when $x \geq 2$



Domain: $x \geq 2$

Range: $y \geq 5$

Interval Notation:

"[" or "]" = inclusive
"(" or ")" = exclusive
∞ always gets "(" or ")"

$a \le x \le b$ $[a, b]$

$a < x \le b$ $(a, b]$

$a \le x < b$ $[a, b)$

$a < x < b$ (a, b)

$x \ge a$ $[a, \infty)$

$x > a$ (a, ∞)

$x \le b$ $(-\infty, b]$

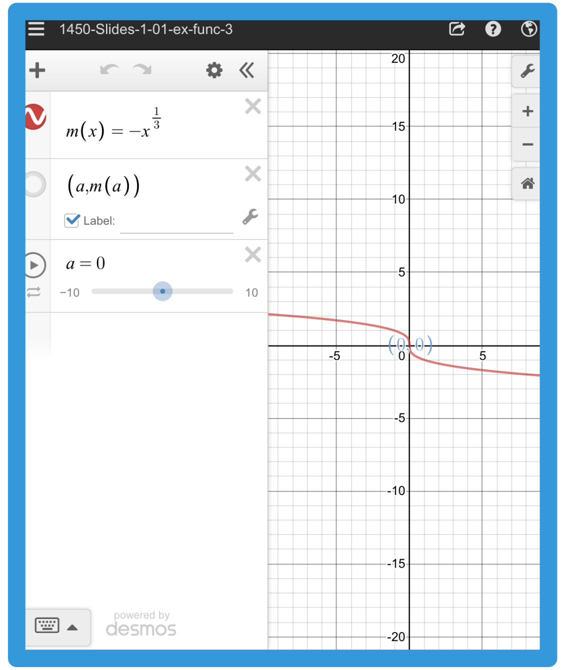
$x < b$ $(-\infty, b)$

Example:

$$m(x) = -\sqrt[3]{x}$$

$$m(8) = -\sqrt[3]{8} = -2$$

$$m(-8) = -\sqrt[3]{-8} = -(-2) = 2$$

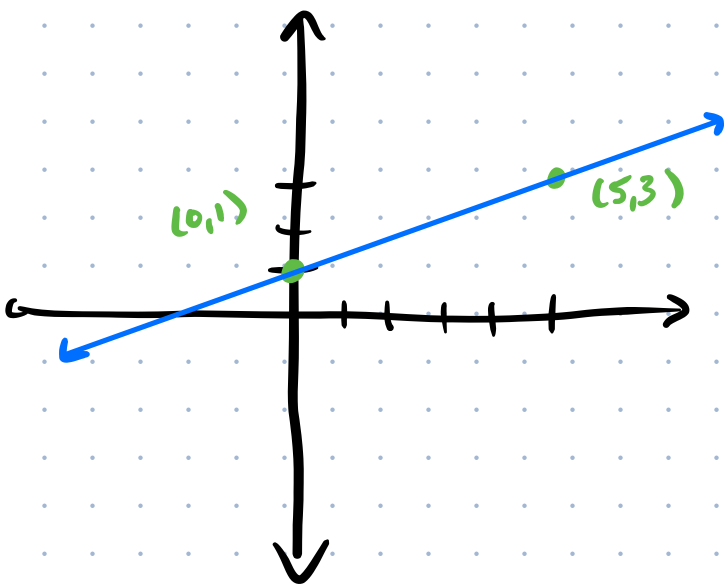


Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Linear Functions (aka, straight lines)

Every line has a slope - how fast it goes up or down

and a y-intercept - where it crosses the y-axis



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3-1}{5-0} = \frac{2}{5}$$

y-int = 1

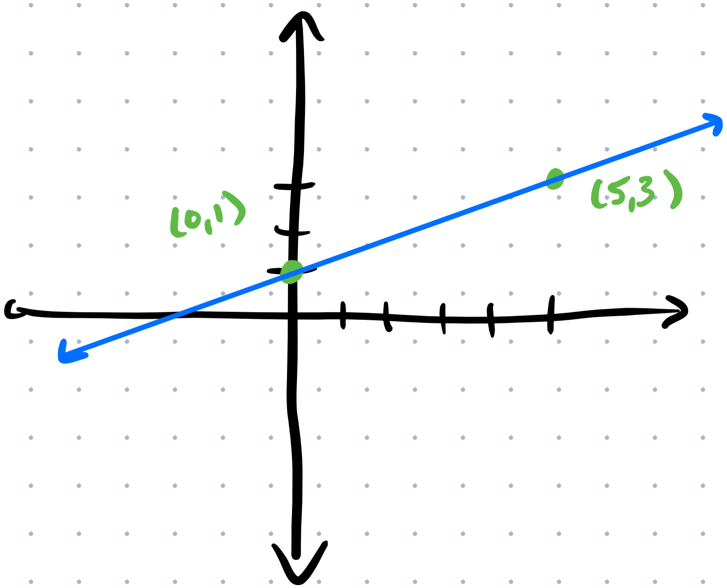
Slope - intercept

A **linear function** has the form

$$y = f(x) = b + mx. \quad = mx + b$$

Its graph is a line such that

- m is the **slope**, or rate of change of y with respect to x.
- b is the **vertical intercept**, or value of y when x is zero.



slope $= \frac{2}{5} = m$

y-int : $1 = b$

$$y = \frac{2}{5}x + 1$$

A line defined by the slope and a point

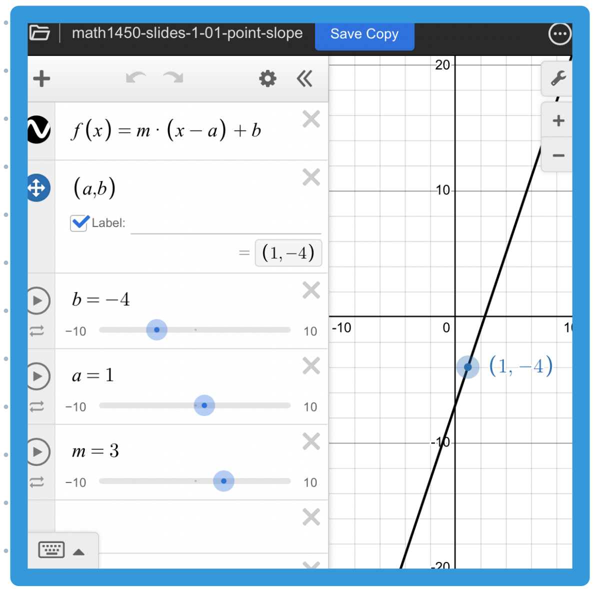
Let y be the line with <u>slope m</u> that passes through the point (a,b).

$$y-b=m\cdot(x-a)$$

Then: $$y=m\cdot(x-a)+b$$

<u>Ex</u>: The equation for the line with slope 3 that passes through $(\underset{a}{1},\underset{b}{-4})$ is:

$$y=3\cdot(x-1)-4$$
$$=3x-3-4=3x-7$$



A line defined by two points:

1) Compute the slope
2) Use the formula on the previous slide with the slope and either point

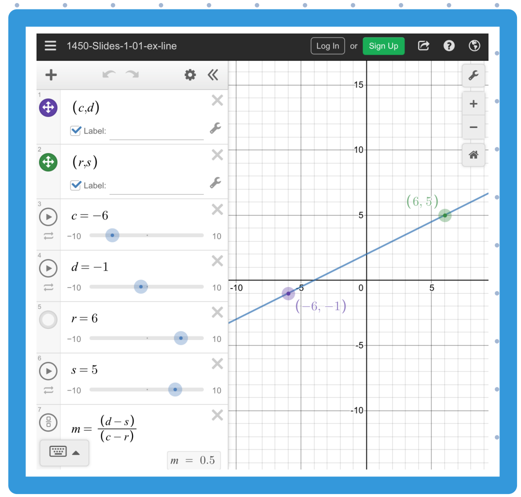
Example:

Let $f(x)$ be the line through points $(1,1)$ and $(4,3)$.

$$\text{slope} = \frac{3-1}{4-1} = \frac{2}{3}$$

$$y = \frac{2}{3}\cdot(x-1)+1$$

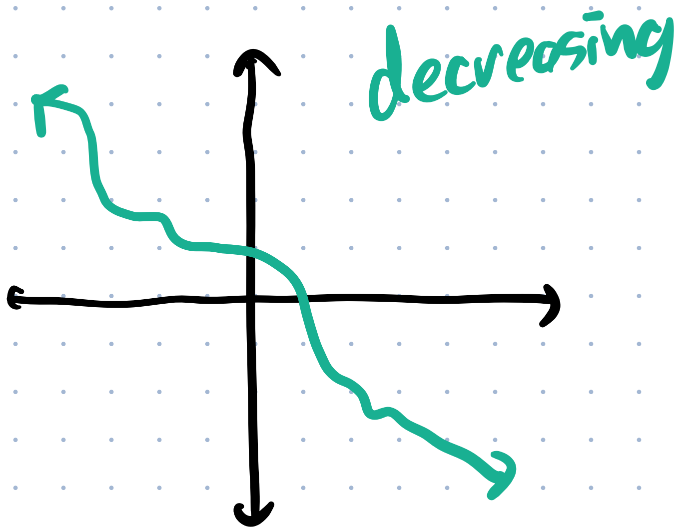
$$= \frac{2}{3}x - \frac{2}{3} + 1 = \boxed{\frac{2}{3}x + \frac{1}{3}}$$

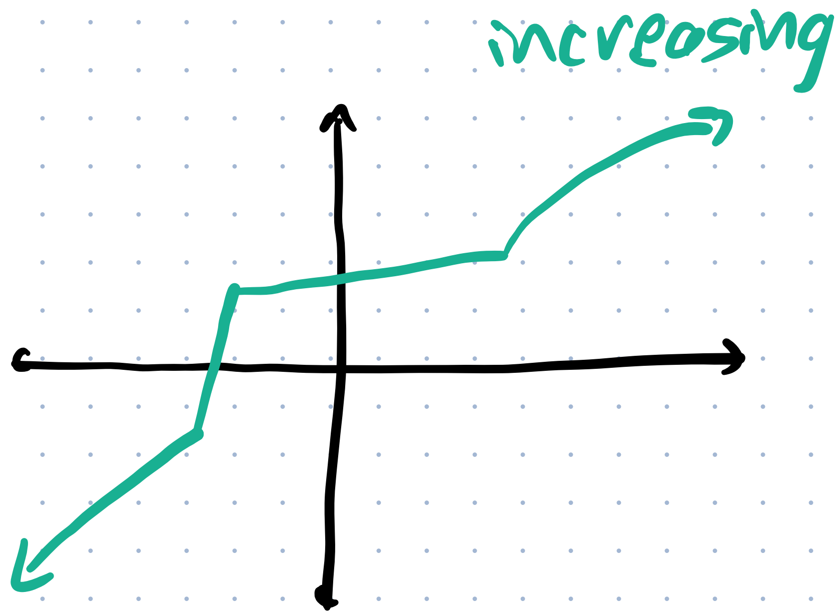


Increasing/Decreasing Functions

* A function is increasing if it always goes up from left to right.

* A function is decreasing if it always goes down from left to right.





For every topic we'll cover, the book has more examples and more detail, so it's a great resource for practice!

Skipping for now: proportionality

Suggested HW

* Every day I will assign suggested homework from the textbook.

* Not collected, not graded, but you can absolutely ask about those problems in office hours, at the help desk, and in discussion.

* The Wiley Plus problems are not enough practice on their own! Most people will need to do some or all of the suggested homework to master the material.

Suggested Textbook HW:

1.1: 4, 5, 8, 9, 11, 12, 18, 35–38, 39–42, 45, 55

Section 1.2 - Exponential Functions

Population of Burkina Faso

Year	Population (millions)	Change in population (millions)
2007	14.235	
		0.425
2008	14.660	
		0.435
2009	15.095	
		0.445
2010	15.540	
		0.455
2011	15.995	
		0.465
2012	16.460	
		0.474
2013	16.934	

$$\frac{2008 \text{ pop}}{2007 \text{ pop}} \approx 1.03$$

$$\frac{2009 \text{ pop}}{2008 \text{ pop}} \approx 1.03$$

$$\frac{2010}{2009} \approx 1.029$$

$$\frac{2011}{2010} \approx 1.029$$

Growing at about 3% per year

Linear Growth - changes by a constant <u>amount</u>
(additive factor)

<u>Ex:</u> 2, 6, 10, 14, 18, ...   +4 each time

Exponential Growth - changes by a constant <u>percentage</u>
(multiplicative factor)

<u>Ex:</u> 2, 8, 32, 128, 512, ...   x4 each time

Back to Burkina Faso – How can we devise a function that models this data?

Consider $P(t)$ to be the function for the population t years after 2007, in millions.

$$P(0) = 14.235$$

$$P(1) = 14.660 \approx 14.235 \cdot 1.03$$

$$P(2) = 15.095 \approx 14.660 \cdot 1.03 = 14.235 \cdot (1.03)^2$$

$$P(3) = 15.540 \approx 14.235 \cdot (1.03)^3$$

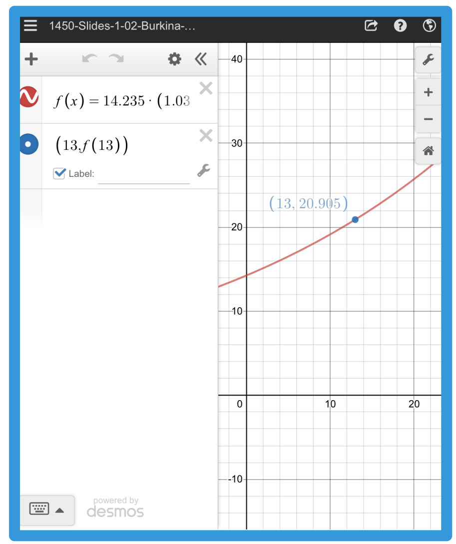
$$P(t) = 14.235 \cdot (1.03)^t$$

What does this predict about 2025? $\underline{P(t) = 14.235 \cdot (1.03)^t}$

$P(2025)$ X

$P(18) = 14.235 \cdot (1.03)^{18} \approx 24.234$

Actual population = 24.157m



Exponential Growth Formula:

$$P(t) = \underbrace{14.235}_{\substack{\text{starting value} \\ \text{y-intercept}}} \cdot (\underbrace{1.03}_{\text{base}})^{\overset{\text{independent variable}}{t}}$$

"growth rate"

$= \text{base} - 1$

$1.03 - 1 = 0.03$

The general form for exponential growth is

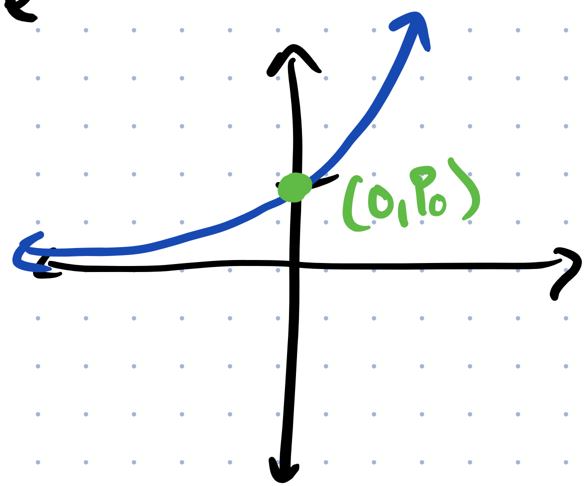
$$P(t) = P_0 \cdot a^t$$

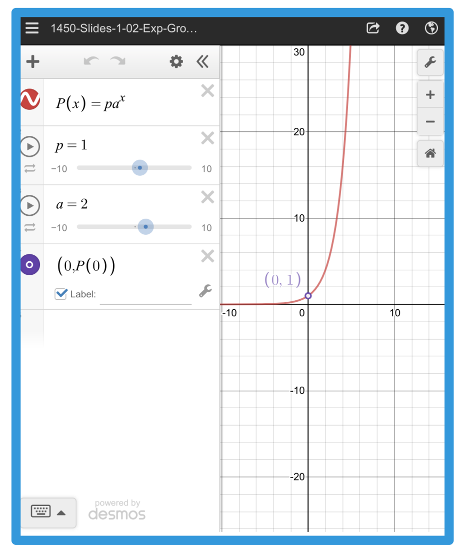
base

growth rate $= a - 1$

For $P(t)$ to <u>grow</u>, we need $a > 1$ and $P_0 > 0$.

General shape:

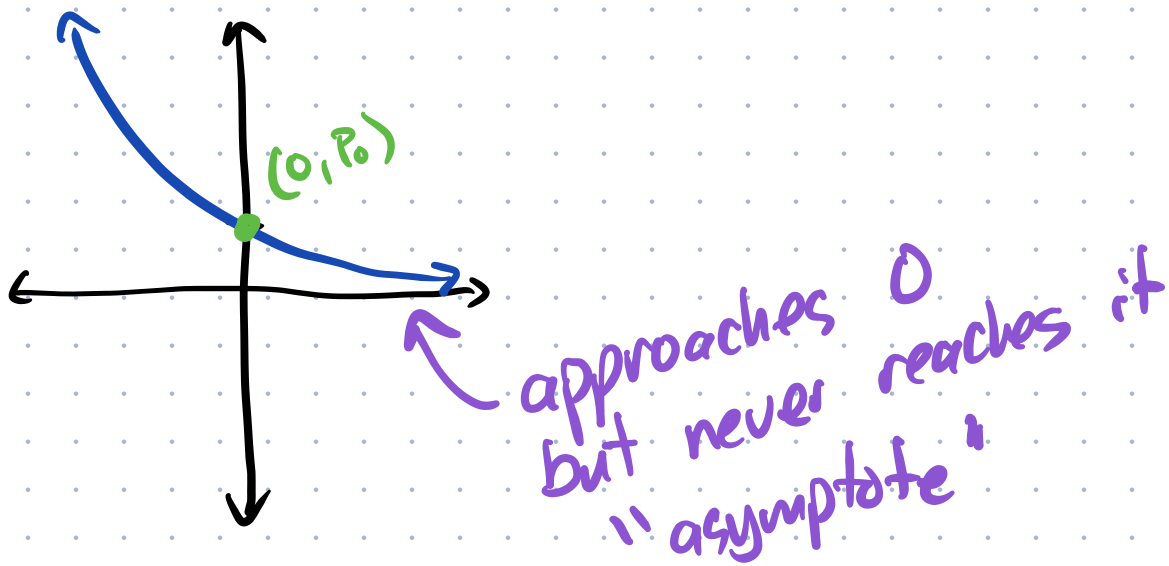




Exponential Decay

$P(t) = P_0 a^t$

When $0 < a < 1$, we have exponential decay



Example:

Your body filters mediation from your blood at a rate that depends on the medication. Ampicillin is filtered at a rate of 60% per hour.

60% gone = 40% is left

Suppose you start with 250 mg in your blood, and let $f(t)$ be the function for the amount left after t hours.

$$f(t) = 250 \cdot (0.4)^t$$

growth rate = -0.6