As a reminder, The final exam will be held on **Wednesday December 10** from **8-10pm** in **Weasler Auditorium**. The exam is cumulative and will cover sections 1.1-1.9, 2.2-2.6, 3.1-3.7, 3.9, 4.1-4.3, 4.6-4.7and 5.1-5.4. Please refer to the previous study guides for practice problems for all sections *except* 4.7and 5.1-5.4, whose review problems can be found below.

To access the review exercises below, you'll need to log in to Wiley and go to the tab marked "Wiley Course Resources". Then, for the Chapter 5 exercises, click the down arrow next to "Ch 5: Key Concept: The Definite Integral (21)". The second item should be "Review Material and Projects: Chapter 05". Clicking this will bring up the review exercises as a downloadable pdf.

The answers to all of the review questions will be posted in a separate pdf.

Of course any other material you wish to study is also a good idea (old homework, quizzes, textbook, lecture notes, etc.).

**"New" suggested review problems from the textbook**

**Ch 4 Rev Ex's** # 82-85

**Ch 5 Rev Ex's** # 1-3, 5-16, 23, 27, 30, 31, 34, 35, 39, 42-47, 50-52, 55-57, 59

**"New" Review Topics**

§4.7 & Chapter 5

- l'Hôpital's Rule and indeterminate limits (§4.7)
- The area under a velocity curve gives the total change in distance (§5.1)
- Definition of the definite integral (§5.2)
- Compute the definite integral from a graph (§5.2)
- Estimate the definite integral using right- and left-hand Riemann sums (§5.1-5.2)
- The Fundamental Theorem of calculus (§5.3)
- Using the Fundamental Theorem to evaluate integrals (§5.3)
- Combining definite integrals (§5.4)
- Area between curves (§5.4)
- Comparison of definite integrals (§5.4)
- Average value of a function (§5.4)

**Additional Review Problems**

**1.** Determine whether L'Hopital's Rule applies for the limit. Explain your reasoning. Evaluate the limit.

(a) $\displaystyle\lim_{x\to\infty}\frac{3x^2}{1-4x^2}$

(b) $\displaystyle\lim_{x\to 0}\frac{3x^2}{1-4x^2}$

(c) $\displaystyle\lim_{x\to 1}\frac{2^x-8}{x-3}$

(d) $\displaystyle\lim_{x\to 3}\frac{2^x-8}{x-3}$

**2.** Use L'Hopital's Rule to evaluate the limit, if possible.

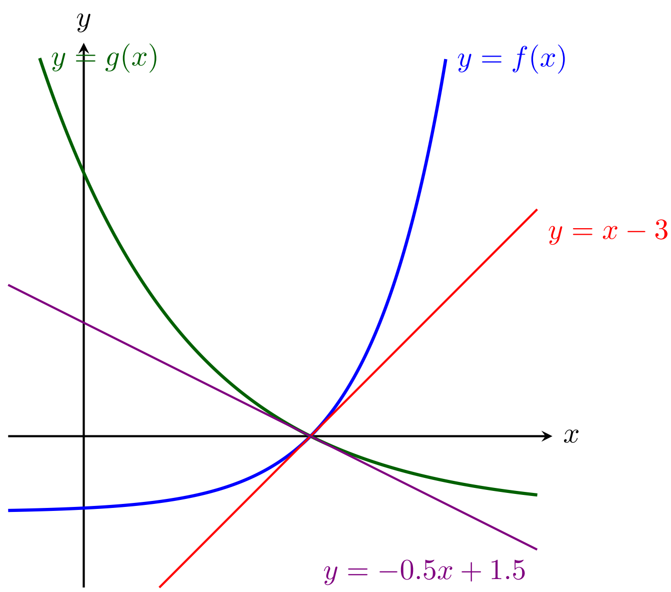
(a) $\displaystyle\lim_{x\to 0}\frac{4e^{-x}-4}{x}$

(b) $\displaystyle\lim_{x\to \frac{\pi}{2}}\frac{2x-\pi}{\cos(x)}$

(c) $\displaystyle\lim_{x\to\infty}\frac{3x+e^{2x}}{1+x^2+3e^{2x}}$

(d) $\displaystyle\lim_{t\to 1}\frac{\ln(t)}{t^2-1}$

**3.** The functions $y=f(x)$ and $y=g(x)$ and their tangent lines at $x=3$ are shown below. Find $\displaystyle\lim_{x\to 3}\frac{f(x)}{g(x)}$.




**4.** Determine the indeterminate form of the limit. Then evaluate the limit.

(a) $\displaystyle\lim_{t\to 0^+}\left(\frac{1}{t}-\frac{1}{e^t-1}\right)$ Hint. Write $\displaystyle\frac{1}{t}-\frac{1}{e^t-1}=\frac{(e^t-1)-t}{t(e^t-1)}$.

(b) $\displaystyle\lim_{x\to 0^+}(1+x)^{1/x}$ Hint. Consider $\displaystyle\lim_{x\to 0^+}\ln(1+x)^{1/x}$.

**5.** The velocity of a moving object is increasing from time $t=0$ seconds to $t=30$ seconds. The object's velocity is measure every 5 seconds.

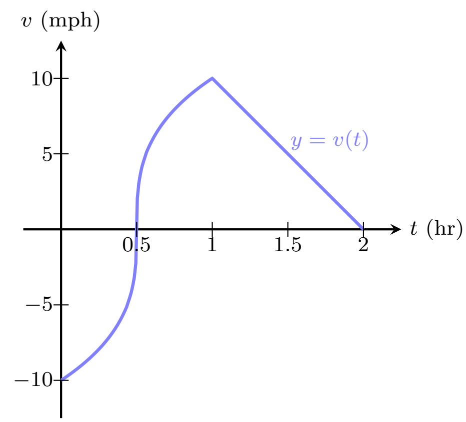
| $t$ (sec) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| $v(t)$ (ft/sec) | 0 | 10 | 16 | 25 | 32 | 45 | 51 |

(a) Represent the total distance the object traveled using a definite integral.

(b) Find an upper estimate of the total distance traveled. Indicate $n$ and $\Delta t$.

(c) Find lower estimate of the total distance traveled. Indicate $n$ and $\Delta t$.

**6.** The velocity $v = v(t)$ (in mph) of a bicyclist at time $t$ hours is given by the below graph.

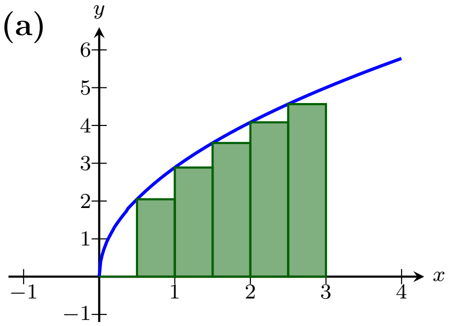



(a) Shade the area associated with the definite integral $\int_0^2 v(t)\,dt$. Explain the meaning of the area above the $t$-axis and area below the $t$-axis. Explain the meaning of $\int_0^2 v(t)\,dt$.

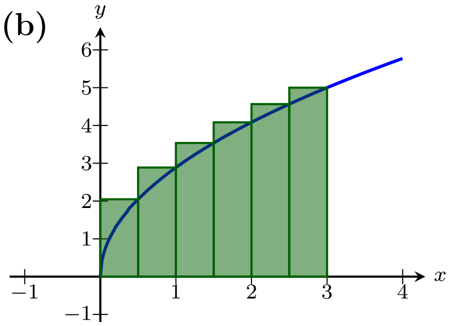
(b) Find the exact time $t_0$ when the bicyclist has returned to their starting point. Explain your reasoning.

**7.** The below figures represent Riemann sums to approximate $\int_a^b f(x)\,dx$ with $n$ subdivisions. Answer both (i) and (ii) for each figure.

**(a)**




**(b)**




(i) Is the figure for a left-hand Riemann sum or a right-hand Riemann sum? What are $a$, $b$, $n$ and $\Delta x$?

(ii) Is the Riemann sum an over approximation or under approximation of $\int_a^b f(x)\,dx$. Explain.

**8.** Shade the area associated with the definite integral $\int_0^2 \frac{1}{x+1}\,dx$.

(a) Estimate $\int_0^2 \frac{1}{x+1}\,dx$ using a left-hand sum with $n = 4$ rectangles. Is this an over or under estimation? Explain.

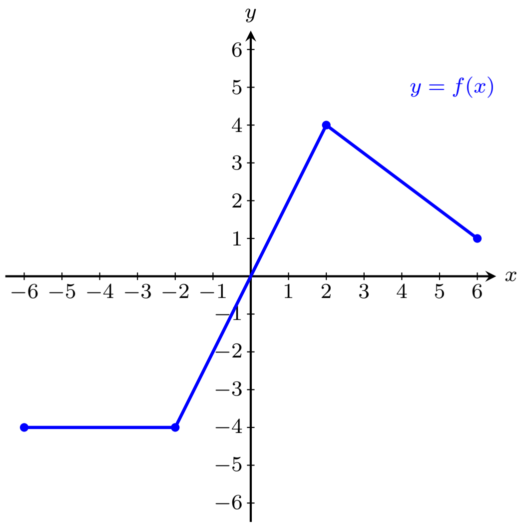
(b) Repeat with a right-hand sum with $n = 4$ rectangles.

**9.** Use the table to estimate $\int_0^{100} f(x)\,dx$ with a right-hand sum. Repeat with a left-hand sum.

| $x$ | 0 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|
| $f(x)$ | 14 | 19 | 25 | 30 | 32 | 41 |

**10.** Shade the area associated with the definite integral $\int_0^3 x - 2\,dx$. Is $\int_0^3 x - 2\,dx$ positive, negative or zero? Explain without directly computing the integral.

**11.** Use the graph of the function to $y = f(x)$ to find the exact value of the definite integral.




(a) $\int_{-6}^{-2} f(x)\,dx =$

(b) $\int_{-6}^{0} f(x)\,dx =$

(c) $\int_{-2}^{2} f(x)\,dx =$

(d) $\int_{0}^{3} f(x)\,dx =$

(e) $\int_{3}^{6} f(x)\,dx =$

(f) $\int_{-6}^{6} f(x)\,dx =$

**12.** Let $f(x) = F'(x)$. Write $\int_a^b f(x)\,dx$ and evaluate using the Fundamental Theorem of Calculus. Be precise with notation.

(a) $F(x) = \frac{1}{3}x^3$, $a = -1$, $b = 2$

(b) $F(x) = e^{7x}$, $a = 0$, $b = 3$

(c) $F(x) = 2\ln(x)$, $a = 1$, $b = 4$

(d) $F(x) = 3^x$, $a = 0$, $b = 1$

**13.** Let $F(x) = e^{x^2}$.

(a) Find $F'(x)$.

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 2xe^{x^2}\,dx$. Be precise with the notation.

**14.** Evaluate the definite integral $\int_a^b f(x)\,dx$ using the Fundamental Theorem of Calculus. Use interpretation of the definite integral as area between curve $y = f(x)$ and $x$-axis to verify answer is correct.

(a) $\int_0^1 8x\,dx$

(b) $\int_0^{2\pi} \cos(x)\,dx$

**15.** Water is flowing into a leaking rowboat at a rate of $r = r(t)$ liters per minute at $t$ *minutes* from the start of leak.

(a) What are the units on the definite integral $\int_0^{30} r(t)\,dt$? Explain your reasoning.

(b) Explain in words what the definite integral $\int_0^{30} r(t)\,dt$ represents. Include units.

**16.** Pollution is removed for a lake at a rate of $p = p(t)\ \frac{kg}{day}$ on day $t$.

(a) Explain the meaning of $p(30) = 500$. Include units.

(b) Explain in words the meaning of $\int_0^{30} p(t)\,dt = 4000$. Include units on 0, 30 and 4000.

**17.** Suppose $\int_0^5 f(x)\,dx = 4$, $\int_0^5 (f(x))^2\,dx = 8$, $\int_0^5 g(x)\,dx = -3$, and $\int_0^5 (g(x))^2\,dx = 12$. Find the following:

(a) $\int_5^0 f(x)\,dx$

(b) $\int_0^5 6g(x)\,dx$

(c) $\int_0^5 f(x) - g(x)\,dx$

(d) $\int_0^5 3f(x) + 10g(x)\,dx$

(e) $\int_0^5 (f(x))^2 - (g(x))^2\,dx$

(f) $\int_0^5 (g(x))^2\,dx - \left(\int_0^5 g(x)\,dx\right)^2$

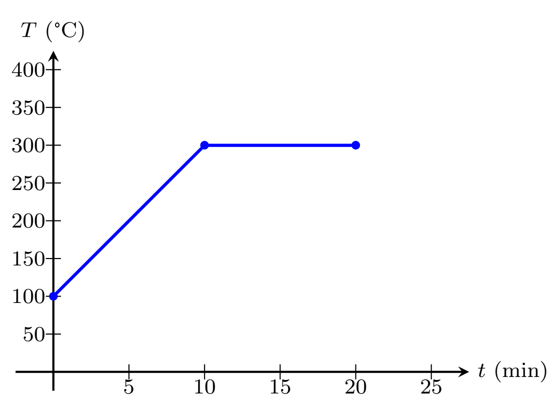
**18.** Suppose $\int_{-2}^3 f(x)\,dx = 1.5$ and $\int_{-2}^8 f(x)\,dx = 4.6$,

(a) Find $\int_3^8 f(x)\,dx$ and $\int_3^8 -2f(x)\,dx$.

(b) Find $\int_{-2}^3 f(x) + 2\,dx$.

**19.** Find the average value of the function $f(x) = 1 + 2x$ over the interval $0 \le x \le 3$.

**20.** The below graph is the temperature $T = f(t)$, in $^\circ F$, of an oven at $t$ minutes. Find the average temperature of the oven from $t = 0$ minutes to $t = 20$ minutes. Include units.



**21.** Sketch the graphs of the two functions and shade the area between the curves. Set up the definite integral that gives the area between the two curves. Evaluate the definite integral.

(i) Under $y = 1 + 2x$ and above $y = 1$ over $[0, 4]$.

(ii) Under $y = 1 - x^2$ and above $y = x^2 - 1$ from $-1 \leq x \leq 1$.