“New” Review Answers

Chapter 4 Review Exercises

1. 0/1; l’Hôpital’s rule does not apply

2. 0/0; l’Hôpital’s rule applies

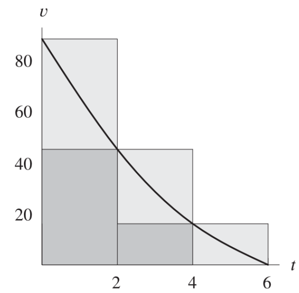
3. 1/2

4. $-1/2$

Chapter 5 Review Exercises

1. **(a)** Lower estimate: 122 feet; Upper estimate: 298 feet

(b)



2. **(a)** 430 ft

(b) (ii)

3. 20

5. 396

6. **(a)** 570 m^3

(b) Every 2 minutes

7. **(a)** Upper estimate: 34.16 m/sec; lower estimate: 27.96 m/sec

(b) New estimate: 31.06 m/sec; The concavity tells us that this should be an overestimate.

8. $\int_{-1}^{1} 4t^3\,dt = 0$

9. $\int_{-2}^{1} 12t^3 - 15t^2 + 5\,dt = -75$

10. $\int_{-2}^{2} 4 - x^2\,dx = 32/3$

11. $\int_{-3}^{3} -(x^2 - 9)\,dx = 36$

12. $\int_{0}^{\pi} \sin(x)\,dx = 2$

13. $\int_{0}^{\pi} 1 - \sin(x)\,dx = \pi - 2$

14. $\int_{0}^{1} x^2 - 5x + 4\,dx + \int_{1}^{3} -x^2 + 5x - 4\,dx = 31/6$

15. $\displaystyle\int_5^7 (\cos(x) + 7) - (\ln(x - 3))\ dx \approx 13.457$

16. $\displaystyle\int_0^2 e^x - e^{2(x-1)}\, dx \approx 2.7622$

23. **(a)** The integral represents the total emissions of nitrogen oxides, in millions of metric tons, during the period from 1970 to 2000.

(b) 772.75 million metric tons

27. **(a)** $\displaystyle\frac{1}{5}\int_0^5 f(x)\, dx$

(b) $\displaystyle\frac{1}{5}\left(\int_0^2 f(x)\, dx - \int_2^5 f(x)\, dx\right)$

30. **(a)** $\ln(t)$

(b) $\displaystyle\int_{10}^{12} \ln(t)\, dt \approx 4.793$

31. **(a)** 0

(b) F increases as b increases

(c) $F(1) \approx 0.747,\ F(2) \approx 0.882,\ F(3) \approx 0.886$

34. 2

35. 3

39. **(a)** The parts above and below the x-axis have the same area, so they cancel out.

(b) ≈ 0.4045

(c) ≈ -0.4049

(d) No; this is because $-3e^{-(-3)^2} \neq 0$, and these were the two different terms between the two sums.

42. $\displaystyle\int_0^T r(t)\, dt = Q$

43. $\displaystyle\int_0^{0.5T} r(t)\, dt > \int_{0.5T}^{T} r(t)\, dt$

44. $\displaystyle\int_0^{T/3} r(t)\, dt > Q/3$

45. $\displaystyle\int_0^{T_h} r(t)\, dt < \int_0^{0.5T} r(t)\, dt$

47. 30/7

50. $\int_7^9 f(t)\,dt = 1$ means that between 7am and 9am, the ice thickened by an additional inch.

51. $F(4) = 2.5$ means that the ice was 2.5 inches thick at 4am.

52. $F'(3.5) = 0.25$ means that at 3:30am, the ice was thickening at a rate of 0.25 inches per hour.

55. ≈ 0.105 miles≈ 554 feet

56. ≈ 0.009 miles≈ 48 feet

57. **(a)** Positive: $0 \le t < 40$, $\approx 59 < t < 60$; Negative: $40 < t \le\approx 59$

(b) ≈ 500 feet at ≈ 42 min

(c) $t \approx 59.5$

(d) $t \approx 41$

(e) Perhaps the balloon tore, causing it to deflate and accelerate downwards.

(f) The total signed area here was about 280, which means the balloon landed 280 feet higher than it started.

59. **(a)** < **(c)** < **(b)** < **(d)**

Selected Answers for the "New" Additional Review Problems

1. (a) yes; indeterminate form $\frac{\infty}{-\infty}$; $-\frac{3}{4}$; (b) no, why?; 0; (c) no, why?; 3; (d) yes; indeterminate form $\frac{0}{0}$; $8\ln(2)$.

2. (a) indeterminate form $\frac{0}{0}$; -4; (b) indeterminate form $\frac{0}{0}$; -2; (c) indeterminate form $\frac{\infty}{\infty}$; $\frac{1}{3}$ (d) indeterminate form $\frac{0}{0}$; $\frac{1}{2}$.

3. $\lim_{x\to 3}\frac{f(x)}{g(x)} = \frac{f'(3)}{g'(3)} = \frac{1}{-0.5} = -2$.

4. (a) indeterminate form $\infty - \infty$; $\frac{1}{2}$; (b) indeterminate for 1^{∞}; e.

5. (a) $\int_0^{30} v(t)\,dt$; (b) upper estimate $= v(5)\Delta t + v(10)\Delta t + v(15)\Delta t + v(20)\Delta t + v(25)\Delta t + v(30)\Delta t = 895\ ft$ where $\Delta t = 5$, $n = 6$; (c) lower estimate $= v(0)\Delta t + v(5)\Delta t + v(10)\Delta t + v(15)\Delta t + v(20)\Delta t + v(25)\Delta t = 640\ ft$ where $\Delta t = 5$, $n = 6$.

6. (b) $t_0 = 1\ hour$; why?

7. (a) (i) left-hand Riemann sum; $a = 0$, $b = 3$; $n = 6$; $\Delta x = \frac{b-a}{n} = 0.5$; (ii) under approximation; (b) (i) right-hand Riemann sum; $a = 0$, $b = 3$; $n = 6$; $\Delta x = \frac{b-a}{n} = 0.5$; (ii) over approximation.

8. (a) left-hand sum $= f(0)\Delta x + f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x = \frac{77}{60} \approx 1.2833$ where $\Delta x = 0.5$; over approximation;

(b) right-hand sum $= f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x + f(2)\Delta x = \frac{19}{20} = 0.95$ where $\Delta x = 0.5$; under approximation;

9. $\Delta x = 20$; right-hand sum $= f(20)\Delta x + f(40)\Delta x + f(60)\Delta x + f(80)\Delta x + f(100)\Delta x = 2940$; left-hand sum $= f(0)\Delta x + f(20)\Delta x + f(40)\Delta x + f(60)\Delta x + f(80)\Delta x = 2400$.

10. Negative; more area below x-axis than above x-axis.

11. (a) -16; (b) -20; (c) 0; (d) 8; (e) $\frac{15}{2} = 7.5$; (f) $-\frac{9}{2} = -4.5$.

12. (a) $\int_a^b f(x)\,dx = \int_{-1}^{2} x^2\,dx$; $\int_{-1}^{2} x^2\,dx = \frac{1}{3}x^3\Big|_{x=-1}^{x=2} = 3$;

(b) $\int_a^b f(x)\,dx = \int_0^3 7e^{7x}\,dx$; $\int_0^3 7e^{7x}\,dx = e^{7x}\Big|_{x=0}^{x=3} = e^{21} - 1$;

(c) $\int_a^b f(x)\,dx = \int_1^4 \frac{2}{x}\,dx$; $\int_1^4 \frac{2}{x}\,dx = 2\ln(x)\Big|_{x=1}^{x=4} = 2\ln(4)$;

(d) $\int_a^b f(x)\,dx = \int_0^1 \ln(3)\,3^x\,dx$; $\int_0^1 \ln(3)\,3^x\,dx = 3^x\Big|_{x=0}^{x=1} = 2$.

13. (a) $F'(x) = 2xe^{x^2}$; (b) $\int_0^1 2xe^{x^2}\,dx = e^{x^2}\Big|_{x=0}^{x=1} = e^{1^2} - e^{0^2} = e - 1$.

14. (a) $\int_0^1 8x\,dx = 4x^2\Big|_{x=0}^{x=1} = 4$; (b) $\int_0^{2\pi} \cos(x)\,dx = \sin(x)\Big|_{x=0}^{x=2\pi} = \sin(2\pi) - \sin(0) = 0$.

15. (a) $\frac{liters}{minute} \cdot minutes = liters$; (b) total amount of water (in *liters*) leaked into rowboat over the first 30 *minutes* of the leak.

16. (a) On day 30, pollution is being removed from the lake at a rate of 500 $\frac{kg}{day}$. (b) From *day* 0 to *day* 30, a total of 4000 *kg* of pollution was removed from the lake.

17. (a) -5; (b) $6(-3) = -18$; (c) $4 - (-3) = 7$; (d) $3(4) + 10(-3) = -18$; (e) $8 - 12 = -4$; (f) $12 - (-3)^2 = 3$.

18. (a) $\int_{-2}^3 f(x)\,dx + \int_3^8 f(x)\,dx = \int_{-2}^8 f(x)\,dx$, and so $\int_3^8 f(x)\,dx = 4.6 - 1.5 = 3.1$;

$$\int_3^8 -2f(x)\,dx = -2(3.1) = -6.2;$$

(b) $\int_{-2}^3 f(x) + 2\,dx = \int_{-2}^3 f(x)\,dx + \int_{-2}^3 2\,dx = 1.5 + 10 = 11.5$.

19. average value $= \frac{1}{3-0}\int_0^3 1 + 2x\,dx = \frac{1}{3} \cdot 12 = 4$; Hint. Use area below the curve to compute $\int_0^3 1 + 2x\,dx$; can also use Fundamental Theorem of Calculus with $F(x) = x + x^2$, why?

20. average value $= \frac{1}{20-0}\int_0^{20} f(t)\,dt = \frac{1}{20} \cdot 5000 = 250°F$; Hint. use area below the curve to compute $\int_0^{20} f(t)\,dt$.

21. (b) (i) area between curves $= \int_0^4 [(1 + 2x) - 1]\,dx = \int_0^4 2x\,dx$; Show $\int_0^4 2x\,dx = 16$ using Fundamental Theorem of Calculus with $F(x) = x^2$, why?

(ii) area between curves $= \int_{-1}^1 [(1 - x^2) - (x^2 - 1)]\,dx = \int_{-1}^1 2 - 2x^2\,dx$;

Show $\displaystyle\int_{-1}^{1} 2 - 2x^2 \, dx = \frac{8}{3}$ using Fundamental Theorem of Calculus with $F(x) = 2x - \dfrac{2}{3}x^3$, why?