## Exam 3 Review Answers

### Chapter 3 Review Exercises

**9.** $f'(x) = x^2 \ln(x)$

**11.** $g'(\theta) = e^{\sin(\theta)} \cos(\theta)$

**13.** $f'(w) = -\tan(w-1)$

**14.** $f'(y) = \dfrac{3}{y \ln(2y^3)}$

**17.** $\dfrac{dz}{d\theta} = 3\sin^2(\theta)\cos(\theta)$

**18.** $f'(t) = -6\cos(3t+5)\sin(3t+5)$

**19.** $M'(\alpha) = 6\tan(2+3\alpha)\sec^2(2+3\alpha)$

**20.** $s'(\theta) = 6\sin(3\theta-\pi)\cos(3\theta-\pi)$

**21.** $h'(t) = \dfrac{-e^{-t}-1}{e^{-t}-t}$

**22.** $p'(\theta) = \dfrac{-\theta\cos(5-\theta) - 2\sin(5-\theta)}{\theta^3}$

**23.** $w'(\theta) = \dfrac{\sin(\theta) - 2\theta\cos(\theta)}{\sin^3(\theta)}$

**27.** $h'(z) = \dfrac{1}{\sqrt{\sin(2z)\cos^3(2z)}}$

**28.** $q'(\theta) = \dfrac{4\theta - \sin(2\theta)}{\sqrt{4\theta^2 - \sin^2(\theta)}}$

**29.** $\dfrac{dw}{dz} = -4 \cdot 2^{-4z}\ln(2)\sin(\pi z) + 2^{-4z}\pi\cos(\pi z)$

**30.** $g'(t) = \dfrac{3}{1+(3t-4)^2}$

**38.** $g'(x) = \dfrac{6x}{1+9x^4}$

**40.** $F'(x) = a$

**49.** $\dfrac{dy}{dx} = \dfrac{-2}{x^2+4}$

**50.** $r'(t) = \dfrac{\cos(t/k)}{k\sin(t/k)}$

**55.** $f'(x) = \dfrac{\ln(x)}{(1+\ln(x))^2}$

**68.** $N'(\theta) = k$

**76.** $\dfrac{dy}{dx} = \dfrac{8xy - 3x^2}{3y^2 - 4x^2}$

**77.** $\dfrac{dy}{dx} = 0$

**78.** $2/3$

**79.** $-3$

**80.** $y = -\frac{4}{5}x - \frac{8}{5}$; $y = -\frac{1}{5}x + \frac{8}{5}$

**96.** **(f)** $4\ln(3) - \dfrac{16}{3} \approx -0.939$

**97.** **(f)** $y = \left(4\ln(3) - \dfrac{16}{3}\right)(x-2) + 4\ln(3) \approx -0.939x + 6.272$

**106.** **(b)** $f(4.1) \approx 2.024845\ldots$, $L(4.1) = 2.025$

**(c)** $f(16) = 4$, $L(16) = 5$

**107.** **(a)** $a = 2$, $f(a) = 1$, $f'(a) = -3$

**(b)** $f(2.1) \approx 0.7$, underestimate; $f(1.98) \approx 1.06$, overestimate

## Chapter 4 Review Exercises

**1.** Local min at $x \approx 1.2, 6$; Local max at $x \approx 0.5, 3.5, 6.5$; Global max at $x \approx 3.5$; Global min at $x \approx 6$

**2.** Local min at $x \approx 1.8$; Local max at $x \approx 0.7, 5$; Critical point at $x \approx 4$; Global max at $x \approx 5$; Global min at $x \approx 1.8$

**3.** **(a)** $f'(x) = 3x^2 - 6x$; $f''(x) = 6x - 6$

**(b)** $x = 0, 2$

**(c)** $x = 1$

**(d)** $f(-1) = -4$, $f(0) = 0$, $f(2) = -4$, $f(3) = 0$; Global max is 0 at $x = 0, 3$, Global min is $-4$ at $x = -1, 2$

**4.** **(a)** $f'(x) = 1 + \cos(x)$; $f''(x) = -\sin(x)$

**(b)** $x = \pi$

(c) $x = \pi$

(d) $f(0) = 0$, $f(\pi) = \pi$, $f(2\pi) = 2\pi$; Global max is $2\pi$ at $x = 2\pi$, Global min is 0 at $x = 0$

5. (a) $f'(x) = e^{-x}(\cos(x) - \sin(x))$; $f''(x) = -2e^{-x}\cos(x)$

(b) $x = \pi/4, 5\pi/4$

(c) $x = \pi/2, 3\pi/2$

(d) $f(0) = 0$, $f(\pi/4) = \dfrac{1}{e^{\pi/4}\sqrt{2}}$, $f(5\pi/4) = -\dfrac{1}{e^{5\pi/4}\sqrt{2}}$, $f(2\pi) = 0$; Global max is $\dfrac{1}{e^{\pi/4}\sqrt{2}}$ at $x = \pi/4$; Global min is $-\dfrac{1}{e^{5\pi/4}\sqrt{2}}$ at $x = 5\pi/4$

6. (a) $f'(x) = -\dfrac{2}{3}x^{-5/3} + \dfrac{1}{3}x^{-2/3}$, $f''(x) = \dfrac{10}{9}x^{-8/3} - \dfrac{2}{9}x^{-5/3}$

(b) $x = 2$

(c) None

(d) $f(1.2) \approx 1.948$, $f(2) \approx 1.890$, $f(3.5) \approx 1.952$; Global max is $\approx 1.952$ at $x = 3.5$; Global min is $\approx 1.890$ at $x = 2$

7. (a) $f'(x) = 6x^2 - 18x + 12$; $f''(x) = 12x - 18$

(b) $x = 1, 2$

(c) $x = 3/2$

(d) $\lim_{x\to-\infty} f(x) \to -\infty$, $f(1) = 6$, $f(2) = 5$, $\lim_{x\to\infty} f(x) \to \infty$; There is neither a global max nor a global min

8. (a) $f'(x) = \dfrac{8x}{(x^2+1)^2}$; $f''(x) = \dfrac{8 - 24x^2}{(x^2+1)^3}$

(b) $x = 0$

(c) $x = \pm 1/\sqrt{3}$

(d) $\lim_{x\to-\infty} f(x) = 4$, $f(0) = 0$, $\lim_{x\to\infty} = 4$; Global min is 0 at $x = 0$ and there is no global max

9. (a) $f'(x) = e^{-x}(1 - x)$; $f''(x) = e^{-x}(x - 2)$

(b) $x = 1$

(c) $x = 2$

(d) $\lim_{x\to-\infty} f(x) \to -\infty$, $f(1) = 1/e$, $\lim_{x\to\infty} f(x) = 0$; Global max is $1/e$ at $x = 1$ and there is no global min

10. Global max is $\dfrac{1}{e^{\pi/4}\sqrt{2}}$ at $x = \pi/4$, global min is $-\dfrac{1}{e^{5\pi/4}\sqrt{2}}$ at $x = 5\pi/4$

11. Global max is $e^{\pi} - 1$ at $x = \pi$, global min is 2 at $x = 0$

**12.** Global max is 16 at $x = 3$, global min is 1 at $x = 0$

**13.** Global max is 1 at $x = 0$, global min is $e^{-100}$ at $x = 10$

**14.** Global min is 3 at $x = 1/2$ and there is no global max

**15.** Global max is 1 at $t = 0$ and there is no global min

**16.** Global max is $1/2$ at $x = 1$ and there is no global min

**17.** Local max at $x = -3$, local min at $x = 1$, inflection point at $x = -1$

**18.** Local max at $x = -3$, local min at $x = 3$, inflection points at $x = 0, \pm 3/\sqrt{2}$

**19.** Local min at $x = 2$

**20.** Local max at $x = 0$, inflection points at $x = \pm 1/\sqrt{2}$

**21.** Local max at $x = -2/5$, local min at $x = 0$, inflection points at $x = (-2 \pm \sqrt{2})/5$

**22.** Local min at $x = 0$, inflection points at $x = \pm 1/\sqrt{3}$

**24.** $t = (1/2)\ln(6)$

**27.** **(a)** Critical points: $(0, b)$, $(\sqrt{a}, b - a^2)$, $(-\sqrt{a}, b - a^2)$; Inflection points: $\left(\sqrt{\frac{a}{3}}, b - \frac{5a^2}{9}\right)$, $\left(-\sqrt{\frac{a}{3}}, b - \frac{5a^2}{9}\right)$

**(b)** $a = 4$, $b = 21$

**(c)** $\left(\sqrt{\frac{4}{3}}, \frac{109}{9}\right)$, $\left(-\sqrt{\frac{4}{3}}, \frac{109}{9}\right)$

**30.** 32,400

**31.** 18

**34.** 1/4

**35.** 2.5 g/hr

**36.** 0.6 $\text{cm}^2$/min

**37.** **(a)** $x_3$

**(b)** $x_1, x_5$

**(c)** $x_2$

**(d)** 0

**38.** Local max at $x = 1$, local min at $x = 3$

**39.** Local min at $x \approx -1.2$, critical point (neither max nor min) at $x = 1$

**40.** Answers will vary

**48.** $x = 2/\sqrt{3}$ cm, $h = 2/\sqrt{3}$ cm

**49.** $x = 2\sqrt{2/3}$ cm, $h = \sqrt{2/3}$ cm

**50.** $r = 2/\sqrt{3\pi}$ cm, $h = 4/\sqrt{3\pi}$ cm

**51.** $x = \sqrt{8/3\pi}$, $h = \sqrt{8/3\pi}$ cm

**52.** $\dfrac{-e^{-5\pi/4}}{\sqrt{2}} \leq e^{-x}\sin(x) \leq \dfrac{e^{-\pi/4}}{\sqrt{2}}$ for $x \geq 0$

**54.** $5 \leq x^3 - 6x^2 + 9x + 5 \leq 25$ for $0 \leq x \leq 5$

**65.** Area is 1; vertices are $(\pm\sqrt{1/2}, 0)$, $(\pm\sqrt{1/2}, \sqrt{1/2})$

**66.** Area is $e^{-3}/2$; $x = 1$

**67.** $(x, y) \approx (0.59, 0.35)$

**68.** $(x, y) \approx (1, 1)$

**69.** $r = \sqrt{\dfrac{2A}{\pi + 4}}$, $h = \sqrt{\dfrac{2A}{\pi + 4}}$

**70.** Minimum cost$\approx$ \$8378.54

**71.** 40 ft by 80 ft

**72.** **(a)** 150 passengers; price is \$1,500 per passenger

**(b)** Maximum profit is \$89,000; 130 passengers paying \$1,700 per passenger

**75.** The minimum distance will be $\approx 331$ meters, so the ships do not need to change course.

**81.** $dV/dt \approx -0.0545$ m$^3$ per minute

**89.** $1/\pi$ $\mu$m per day

**90.** 2 cm per sec

**92.** $\dfrac{dD}{dt} = -Kre^{-r}\dfrac{dr}{dt}$

**94.** $0.27\pi$ cm$^3$ per hour

**95.** $\dfrac{dh}{dt} = \dfrac{3}{10\pi h^2}$, $\dfrac{dr}{dt} = \dfrac{\sqrt{3}}{10\pi h^2}$

**96.** **(a)** $\sqrt{20h - h^2}$

**(b)** $1/10\sqrt{3}$ cm per hour

**97.** **(a)** $\dfrac{dF}{dt} = \dfrac{K}{a^3}\dfrac{dy}{dt}$

**(b)** $\dfrac{dF}{dt} = 0$

**(c)** $\dfrac{dF}{dt} = -\dfrac{7K}{5^{5/2}a^3}\dfrac{dy}{dt}$

**98.** $-10$ ohms/min

**99.** $2580/\sqrt{42}$ mph

## Selected Answers for the Additional Review Problems

**1.** (a) $y' = 4\cos(t) - 10 \cdot \dfrac{1}{\cos^2(t)}$ or $y' = 4\cos(t) - 10\sec^2(t)$;

(b) $f(x) = 8x^{-4} - \dfrac{1}{8}\cos(4x)$; $f'(x) = 8(-4)x^{-5} - \dfrac{1}{8}(-\sin(4x) \cdot 4) = -32x^{-5} + \dfrac{1}{2}\sin(4x)$;

(c) $y = 6[\tan(\theta)]^3$; $\dfrac{dy}{d\theta} = 6{\cdot}3[\tan(\theta)]^2{\cdot}\dfrac{1}{\cos^2(\theta)} = \dfrac{18[\tan(\theta)]^2}{\cos^2(\theta)}$ or $\dfrac{dy}{d\theta} = 6{\cdot}3[\tan(\theta)]^2{\cdot}\sec^2(\theta)$;

(d) $$P' = \frac{(1-5\tan(r)) \cdot 5\dfrac{1}{\cos^2(r)} - (1+5\tan(r)) \cdot (-5)\dfrac{1}{\cos^2(r)}}{(1-5\tan(r))^2}$$
$$= \frac{\dfrac{5(1-5\tan(r))}{\cos^2(r)} - \dfrac{-5(1+5\tan(r))}{\cos^2(r)}}{(1-5\tan(r))^2}$$
or
$$P' = \frac{(1-5\tan(r)) \cdot 5\sec^2(r) - (1+5\tan(r)) \cdot (-5)\sec^2(r)}{(1-5\tan(r))^2};$$

(e) $y' = x^7 \cdot (-\sin(4x) \cdot 4) + \cos(4x) \cdot 7x^6 = -4x^7\sin(4x) + 7x^6\cos(4x)$;

(f) $\dfrac{dy}{dt} = \cos(\cos(7t)) \cdot (-\sin(7t)) \cdot 7 = -7\cos(\cos(7t))\sin(7t)$;

(g) $h'(x) = 10e^{2\sin(\pi x)} \cdot (2\cos(\pi x) \cdot \pi) = 10\pi\cos(\pi x)e^{2\sin(\pi x)}$;

(h) $\ell(t) = [\sin(6t)]^{5/3}$; $\ell'(t) = \dfrac{5}{3}[\sin(6t)]^{2/3} \cdot (\cos(6t) \cdot 6) = 10[\sin(6t)]^{2/3}\cos(6t)$;

(i) $s' = -\sin(t^2e^t)[t^2 \cdot e^t + e^t \cdot 2t] = -\sin(t^2e^t)[t^2e^t + 2te^t]$;

(j) $\dfrac{dy}{d\theta} = 1.2\theta^{0.2} - 0.5e^{\tan(\theta^3)} \cdot (\sec^2(\theta^3) \cdot 3\theta^2) = 1.2\theta^{0.2} - 1.5e^{\tan(\theta^3)}\sec^2(\theta^3)$.

**2.** $f'(x) = -\sin(\pi x^2) \cdot 2\pi x = -2\pi x\sin(\pi x^2)$;
$f''(x) = -2\pi x \cdot (\cos(\pi x^2) \cdot 2\pi x) + \sin(\pi x^2) \cdot (-2\pi) = -4\pi^2x^2(\cos(\pi x^2) + (-2\pi)\sin(\pi x^2)$.

**3.** $y = 4(\theta - \dfrac{\pi}{8}) + 1$.

**4.** (a) $G'(x) = \cos(g(x)) \cdot g'(x)$; $G'(\pi) = \cos(g(\pi)) \cdot g'(\pi) = \cos(0) \cdot (-2) = -2$;

(b) $H'(x) = \dfrac{(\cos(x)+2) \cdot g'(x) - (g(x)+2) \cdot (-\sin(x))}{(\cos(x)+2)^2}$;

$$\begin{aligned} H'(\pi) &= \frac{(\cos(\pi)+2) \cdot g'(\pi) - (g(\pi)+2) \cdot (-\sin(\pi))}{(\cos(\pi)+2)^2} \\ &= \frac{(-1+2) \cdot (-2) - (0+2) \cdot (-0)}{(-1+2)^2} \\ &= -2 \end{aligned}.$$

**5.** (a) $y = 2\ln(x^{10}) + 2x^{-10} + 10$; $y' = 2 \cdot \dfrac{1}{x^{10}} \cdot 10x^9 + 2(-10)x^{-11} = \dfrac{20}{x} - 20x^{-11}$;

(b) $y' = x^3 \cdot (\dfrac{1}{9x} \cdot 9) + \ln(9x) \cdot 3x^2 = x^2 + 3x^2 \ln(9x)$;

(c) $f'(x) = \dfrac{1}{x^4+8} \cdot 4x^3 = \dfrac{4x^3}{x^4+8}$;

(d) $g'(t) = \dfrac{\arcsin(t) \cdot 5 - (5t+1) \cdot \frac{1}{\sqrt{1-t^2}}}{(\arcsin(t))^2}$; (e) $y' = 3\dfrac{1}{1+(x^9)^2} \cdot 9x^8 = \dfrac{27x^8}{1+x^{18}}$;

(f) $y' = \sin(4x) \cdot (\dfrac{1}{\sqrt{1-(8x)^2}} \cdot 8) + \arcsin(8x) \cdot (\cos(4x) \cdot 4)$

$= \dfrac{8\sin(4x)}{\sqrt{1-64x^2}} + 4\arcsin(8x)\cos(4x)$;

(g) $\dfrac{dp}{dr} = -\sin(\ln(1+e^r)) \cdot (\dfrac{1}{1+e^r} \cdot e^r) = -\sin(\ln(1+e^r)) \cdot \dfrac{e^r}{1+e^r}$;

(h) $y = (1+\arctan(7t))^{1/2}$;

$y' = \dfrac{1}{2}(1+\arctan(7t))^{-1/2} \cdot (\dfrac{1}{1+(7t)^2} \cdot 7) = \dfrac{1}{2}(1+\arctan(7t))^{-1/2} \cdot (\dfrac{7}{1+49t^2})$;

(i) $v'(x) = -0.5e^{5\arcsin(x^2)} \cdot (5 \cdot \dfrac{1}{\sqrt{1-(x^2)^2}} \cdot 2x) = -0.5e^{5\arcsin(x^2)} \cdot \dfrac{10x}{\sqrt{1-x^4}}$;

(j) $f'(x) = \dfrac{1}{2x+e^{3x}} \cdot (2 + e^{3x} \cdot 3) = \dfrac{2+3e^{3x}}{2x+e^{3x}}$;

(k) $f'(x) = \dfrac{1}{2xe^{3x}} \cdot (2x \cdot (e^{3x} \cdot 3) + e^{3x} \cdot 2) = \dfrac{6xe^{3x} + 2e^{3x}}{2xe^{3x}}$;

or write $f(x) = \ln(2xe^{3x}) = \ln(2) + \ln(x) + \ln(e^{3x}) = \ln(2) + \ln(x) + 3x$,

and so $f'(x) = \dfrac{1}{x} + 3$;

(l) $\dfrac{ds}{dx} = \dfrac{1}{1+(\frac{1}{2x+1})^2} \cdot \dfrac{(2x+1) \cdot 0 - 1 \cdot 2}{(2x+1)^2} = \dfrac{1}{1+(\frac{1}{2x+1})^2} \cdot \dfrac{-2}{(2x+1)^2}$

**6.** $f'(x) = \dfrac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \dfrac{3}{\sqrt{1-9x^2}} = 3(1-9x^2)^{-1/2}$;

$f''(x) = -\dfrac{1}{2}(1-9x^2)^{-3/2} \cdot (-18x) = 9x(1-9x^2)^{-3/2}$.

**7.** (a) slope of tangent line is $m = f'(2) = 1$; (b) $y = 1 \cdot (x-2) + 3$ or $y = x + 1$.

**8.** From graph, $g(1.0) = 3.5$ and $g'(1.0) = \dfrac{4.1 - 3.5}{1.5 - 1.0} = 1.2$; $G'(x) = \dfrac{1}{g(x)} \cdot g'(x)$; $G'(1.0) = \dfrac{1}{g(1.0)} \cdot g'(1.0) = \dfrac{1}{3.5} \cdot 1.2 = \dfrac{12}{35}$.

**9.** (a) write as $x^{-2} + y^2 = 1$; $\dfrac{dy}{dx} = \dfrac{2x^{-3}}{2y}$; (b) $\dfrac{dy}{dx} = \dfrac{-y - 8x}{x - 5e^{5y}}$; (c) $\dfrac{dy}{dx} = \dfrac{\frac{1}{x} - 4x^3y^8}{8x^4y^7 - \frac{1}{y}}$; (d) $\dfrac{dy}{dx} = \dfrac{\frac{7}{1+49x^2}}{2y - \frac{7}{1+49y^2}}$; (e) write as $x^6 = y^3(4y-1)$ or $x^6 = 4y^4 - y^3$; $\dfrac{dy}{dx} = \dfrac{6x^5}{16y^3 - 3y^2}$; (f) $\dfrac{dy}{dx} = \dfrac{2xy^2 - y\cos(xy)}{x\cos(xy) - 2x^2y}$; (g) $\dfrac{dy}{dx} = \dfrac{2x\cos(y) - 2xe^{x^2+y^2}}{2ye^{x^2+y^2} + x^2\sin(y)}$.

**10.** (a) $\dfrac{dy}{dx} = \dfrac{4x^3 + 2xy}{2y - x^2}$; (b) slope is $\left.\dfrac{dy}{dx}\right|_{x=1,y=2} = \dfrac{4(1)^3 + 2(1)(2)}{2(2) - (1)^2} = \dfrac{8}{3}$; (c) $y = \dfrac{8}{3}(x-1) + 2$.

**11.** (a) $f(x) \approx f'(1)(x-1) + f(1)$ for $x \approx 1$, and so $2 + \sqrt[3]{x} \approx \dfrac{1}{3}(x-1) + 3$ for $x \approx 1$; (b) $2 + \sqrt[3]{1.06} \approx \dfrac{1}{3}(1.06 - 1) + 3 = 3.02$; (c) over approximation because tangent line is above the graph of the function $y = f(x)$ at $x = 1.06$.

**12.** $f(x) \approx f'(16)(x-16) + f(16)$ for $x \approx 16$, and so $\dfrac{1}{\sqrt{x}} \approx -\dfrac{1}{128}(x-16) + \dfrac{1}{4}$ for $x \approx 16$; $\dfrac{1}{\sqrt{15.5}} \approx -\dfrac{1}{128}(15.5 - 16) + \dfrac{1}{4} = \dfrac{63}{256} = 0.25390625$.

**13.** $f(x) \approx f'(-1)(x-(-1)) + f(-1)$ for $x \approx -1$, and so $e^{-x^2} \approx 2e^{-1}(x+1) + e^{-1}$ for $x \approx -1$.

**14.** (a) $f(x) \approx f'(1)(x-1) + f(1)$ for $x \approx 1$, and so $2\ln(x) \approx 2(x-1) + 0$ for $x \approx 1$; (b) $2\ln(x) = 0.2 - 0.5x$ becomes $2(x-1) \approx 0.2 - 0.5x$; solving for $x$ yields $x \approx 0.88$.

**15.** (a) $g(x) \approx g'(3)(x-3) + g(3)$ for $x \approx 3$, and so $g(x) \approx -4.25(x-3) + 100$ for $x \approx 3$; (b) $g(3.5) \approx\approx -4.25(3.5-3) + 100 = 97.875$; (c)$g''(3) = -2$ means function is concave down, and so tangent line is above the graph of function $y = g(x)$ near $x = 3$; hence, $g(3) \approx 97.875$ is likely an over approximation. (why?)

**16.** (a) $x = -1$, $x = 2$; (b) $x = -1$, $x = 0$, $x = 1$; (c) $x = -1$, $x = 1$; [$x = 0$ is not a critical point (why?)] (d) $x = -\dfrac{1}{4}$; (e) $x = -\dfrac{1}{b}$; (f) $x = -8$, $x = 0$, $x = 8$.

**18.** (a) (i) $x = -1$, $x = 2$; (ii) increasing on $-\infty < x < -1$ and $2 < x < \infty$; decreasing on $-1 < x < 2$; (iii) local maximum at $x = -1$; local minimum at $x = 2$ (iv) concave down on $-\infty < x < \dfrac{1}{2}$; concave up on $\dfrac{1}{2} < x < \infty$; (v) inflection points at $x = \dfrac{1}{2}$;

(b) (i) $x = -1$, $x = 0$, $x = 1$; (ii) increasing on $0 < x < 1$ and $1 < x < \infty$ (or $0 < x < \infty$); decreasing on $-\infty < x < -1$ and $-1 < x < 0$ (or $-\infty < x < 0$); (iii) local minimum at $x = 1$; neither at $x = -1$ and $x = 1$; no local maximums (iv) concave down on $-1 < x < -\frac{1}{\sqrt{5}}$ and $1 < x < \frac{1}{\sqrt{5}}$; concave up on $-\infty < x < -1$ and $-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$ and $1 < x < \infty$; (v) inflection points at $x = -1$, $x = -\frac{1}{\sqrt{5}}$, $x = \frac{1}{\sqrt{5}}$, $x = 1$;

(c) (i) $x = -\frac{1}{4}$; (ii) increasing on $-\frac{1}{4} < x < \infty$ ; decreasing on $-\infty < x < -\frac{1}{4}$; (iii) local minimum $x = -\frac{1}{4}$; no local maximums (iv) concave down on $-\infty < x < -\frac{1}{2}$; concave up on $-\frac{1}{2} < x < \infty$; (v) inflection point at $x = -\frac{1}{2}$;

(d) (i) $x = -1$, $x = 1$; (ii) increasing on $-1 < x < 0$ and $1 < x < \infty$; decreasing on $-\infty < x < -1$ and $0 < x < 1$; (iii) local minimum $x = -1$ and local minimum at $x = 1$; no local maximums (iv) concave up on $-\infty < x < 0$ and $0 < x < \infty$; (v) no inflection points (why?).

**19.** (a) critical points are $x = -3$, $x = 1$ and $x = 3$; (b) local minimum at $x = 1$; local maximum at $x = 3$; neither at $x = -3$.

**20.** (a) critical points $x = -1$, $x = 2$; $f''(-1) = -18 < 0$, local maximum at $x = -1$; $f''(2) = 18 > 0$, local minimum at $x = 2$;

(b) critical points $x = -3$, $x = 0$, x= 3; $f''(-3) = 72 > 0$, local minimum at $x = -3$; $f''(0) = -36 < 0$, local maximum at $x = 0$; $f''(3) = 72 > 0$, local minimum at $x = 3$;

(c) critical points $x = -1$, $x = 1$; $f''(-1) = 8 > 0$, local minimum at $x = -1$; $f''(1) = 8 > 0$, local minimum at $x = 1$;

(d) critical point $x = -\frac{1}{b}$; $f''(-\frac{1}{b}) = be^{-1} > 0$, local minimum at $x = -\frac{1}{b}$.

**22.** (a) global minimum of $-6$ at $x = 2$; global maximum of 435 at $x = 5$;

(b) global minimum of 0 at $x = -1$; global maximum of 512 at $x = -3$;

(c) global minimum of 0 at $x = 0$; global maximum of $9e^3$ at $x = 3$;

(d) global minimum of $-\frac{2}{3}$ at $x = 1$; global maximum of $\frac{2}{3}$ at $x = -1$ and $x = 8$.

**23.** (a) $x = \frac{3}{2}$ is only critical point in $0 < x < \infty$, and so any local extrema is global extrema at $x = \frac{3}{2}$; $g''(\frac{3}{2}) = \frac{32}{81} > 0$, local minimum at $x = \frac{3}{2}$; conclude global minimum at $x = \frac{3}{2}$.

(b) no global maximum. (why?)

**25.** (a) Maximize $A(\ell) = \ell(20 - \ell)$ over interval $0 < \ell < 20$ where $\ell$ is length of rectangle; Answer: Maximum area is $100\,ft^2$, dimensions are $\ell = 10\,ft$ and $w = 10\,ft$;

(b) Maximize $A(\ell) = \ell(20 - 2\ell)$ over interval $0 < \ell < 10$ where $\ell$ length perpendicular to wall; Answer: Maximum area is $200\,ft^2$, dimensions are $\ell = 20\,ft$ and $w = 10\,ft$.

**26.** Minimize $S(x) = x + \dfrac{784}{x}$ over interval $0 < x < \infty$; Answer: $x = 28$ and $y = 28$ lead to minimal sum of 56;

**27.** Answer: Dose of $d = 400\,milligrams$ maximizes the change in temperature.

**28.** Maximize $V(\ell) = \ell^2(\dfrac{600 - 2\ell^2}{4\ell})$ over interval $0 < \ell < \sqrt{300}$ where $\ell$ is length of the base; Answer: Maximum volume is $V = 1000\,in^3$ when $\ell = 10\,in$.

**29.** Minimize $A(\ell) = (\ell + 2)(\dfrac{256}{\ell} + 4)$ over interval $0 < \ell < \infty$ where $\ell$ is length along top of printed area; Answer: $\ell = 8\,in$ and $w = 16\,in$ of printed region leads to minimum total area of $200\,in^2$ for entire advertisement.

**30.** Minimize $S(r) = 2\pi r^2 + \dfrac{43.312}{r}$ over interval $0 < r < \infty$; Answer: Minimal surface area when radius is $r \approx 1.51\,in$ and height is $h \approx 3.02\,in$.

**31.** (a) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$; $\dfrac{dA}{dt} = \dfrac{3}{2}\pi \approx 4.7124\ \dfrac{mm^2}{day}$; (b) $\dfrac{dr}{dt} = -\dfrac{1}{8\pi} \approx -0.0398\ \dfrac{mm}{day}$.

**32.** (a) $\dfrac{ds}{dt} = 0.0075w^{0.25}h^{-0.25}\dfrac{dh}{dt} + 0.0025w^{-0.75}h^{0.75}\dfrac{dw}{dt}$; $\dfrac{ds}{dt} \approx 0.0121\ \dfrac{m^2}{year}$; (b) $\dfrac{ds}{dt} \approx -0.0024\ \dfrac{m^2}{year}$.

**33.** (a) $h^2 = y^2 + 16$; (b) $\cos(a) = \dfrac{4}{h}$; (c) $2h\dfrac{dh}{dt} = 2y\dfrac{dy}{dt}$; $\dfrac{dy}{dt} = \dfrac{1}{3} \approx 0.3333\ \dfrac{ft}{sec}$; (d) $-\sin(a)\dfrac{da}{dt} = -\dfrac{4}{h^2}\dfrac{dh}{dt}$; $\dfrac{da}{dt} \approx 0.0144\ \dfrac{radians}{sec}$.

**34.** $\dfrac{dV}{dt} = 25\pi h\dfrac{dh}{dt}$; $\dfrac{dh}{dt} = \dfrac{3}{250\pi} \approx 0.0038\ \dfrac{m}{sec}$.

**35.** (a) $r = \sqrt{100 - (10 - h)^2}$ or $r = \sqrt{20h - h^2}$; (b) $\dfrac{dr}{dt} = \dfrac{1}{2}(20h - h^2)^{-1/2}(20 - 2h)\dfrac{dh}{dt}$; $\dfrac{dr}{dt} \approx -0.0980\ \dfrac{cm}{hour}$.