Exam2 Review Answers

Chapter 2 Review Exercises

3. $\dfrac{\ln(3)}{2} \approx 0.5493$

5. 0

7. 0

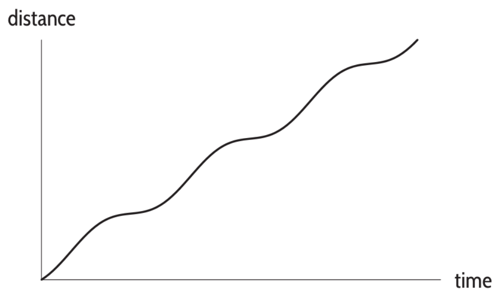
8. **(a)** (i) 8.4

(ii) 8.04

(iii) 8.004

(b) 8

9.



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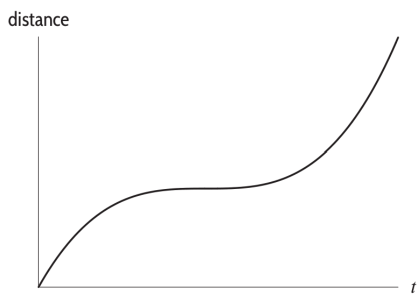

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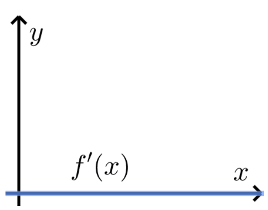

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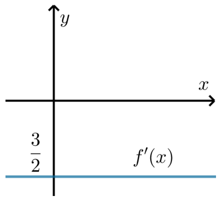

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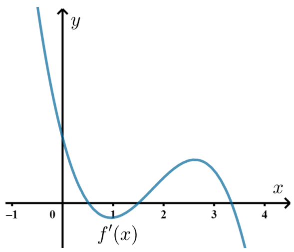

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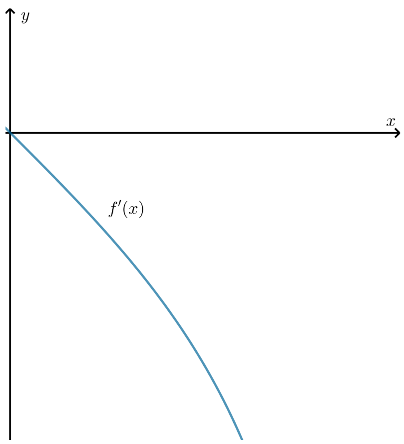

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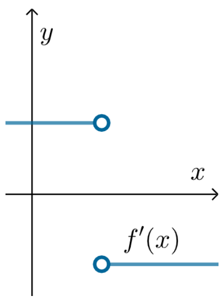

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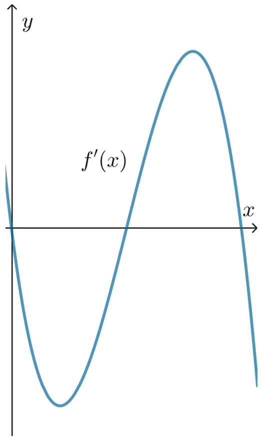

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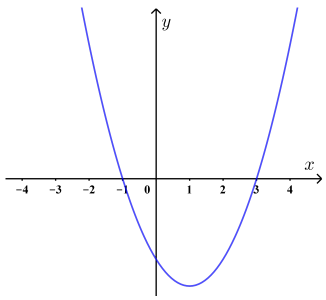
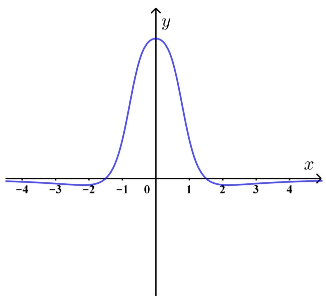


21. 

22. $f'(x) = 10x + 1$

23. $n'(x) = -1/x^2$

24. $f'(3) = 6$; $y = 6x - 8$

29. $f'(x) = 6x$

33. $-2/a^3$

35. $-\dfrac{1}{2\sqrt{a}}$

36. Answers will vary

39. **(a)** A, C, F, H

(b) They are the inflection points of the position function.

41. **(a)** The rate at which the depth is changing in cm/min

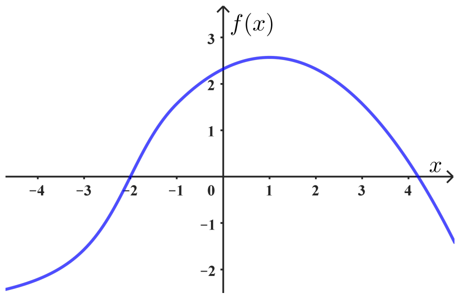
(b) At 30 minutes, the depth is increasing at a rate of 20 cm/min

(c) 12 m/hr

43. **(a)** thousands of dollars per (dollars per gallon); $R'(3)$ tells us how much a \$1 per gallon increase in the price will change the gas station's revenue when the price is \$3 per gallon.

(b) (dollars per gallon) per thousand dollars; $(R^{-1})'(5)$ tells us how much a \$1 increase in revenue will change the price of gasoline when the revenue is \$5,000.

45.



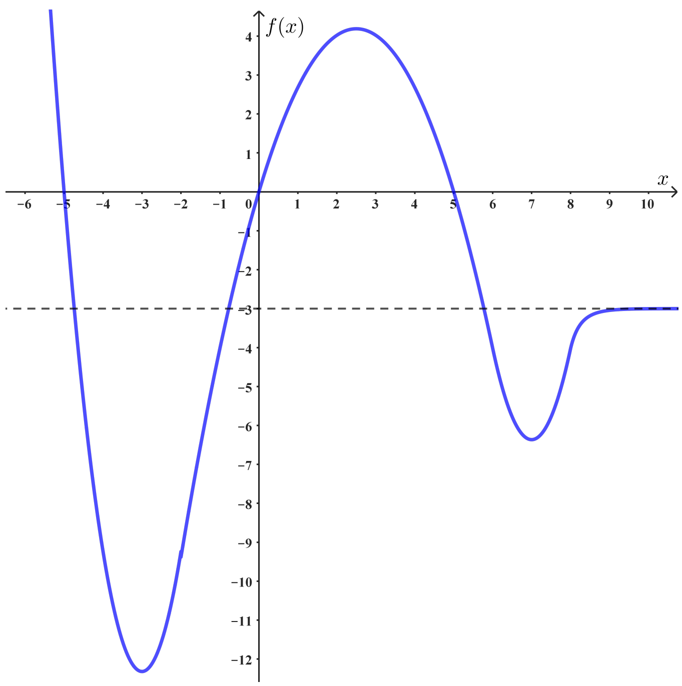
48. $f(22) \approx 357$

50. **(a)** $f(7) = 3$

(b) $f'(7) = 4$

51. $x_1 = 0.9,\ x_2 = 1,\ x_3 = 1.1,\ y_1 = 2.8,\ y_2 = 3,\ y_3 = 3.2$

52.



53. **(a)** Negative

(b) When dw/dt reaches 0, the fire is no longer burning

(c) As the fire increases in vigor, dw/dt will decrease and $|dw/dt|$ will increase.

54. **(a)** Negative; the temperature is decreasing

(b) degrees per minute

55. **(a)** $f(140) = 120$: The dose of the painkiller is 120 mg when the weight of the patient is 140 lbs.

$f'(140) = 3$: If a patient weighs 140 lbs, a 1 lb weight gain would increase the painkiller dose by 3 mg.

(b) $f(145) \approx 135$

68. **(a)** This means that all of $a'(t)$, $a''(t)$, $m'(t)$,, and $m''(t)$ are positive.

(b) Here $2 \leq a'(t) \leq 4$ and $2 \leq m'(t) \leq 10$

Section 2.6

1. **(a)** $x = 1$

(b) $x = 1,\ 3$

2. **(a)** g is continuous for all x values shown

(b) $x = 2,\ 4$

Chapter 3 Review Exercises

1. $\dfrac{dy}{dx} = -12x^3 - 12x^2 - 6$

2. $\dfrac{dy}{dx} = 17 + 12x^{-1/2}$

3. $\dfrac{dw}{dt} = 200t\left(t^2 + 1\right)^{99}$

4. $f'(t) = 3e^{3t}$

5. $\dfrac{dz}{dt} = \dfrac{t^2 + 2t + 2}{(t+1)^2}$

6. $\dfrac{dy}{dt} = \dfrac{-3t^2 + 1}{2\sqrt{t}(t^2+1)^2}$

7. $h'(t) = \dfrac{-8}{(4+t)^2}$

8. $f'(x) = ex^{e-1}$

10. $f'(x) = -\dfrac{4x^2+8x+1}{\left(2+3x+4x^2\right)^2}$

12. $\dfrac{dy}{d\theta} = 1$

15. $g(x) = kx^{k-1} + k^x \cdot \ln(k)$

16. $\dfrac{dy}{dx} = 0$

24. $f'(\theta) = \dfrac{e^{-\theta}}{(1+e^{-\theta})^2}$

25. $g'(w) = \dfrac{-2^w \ln(2) - e^w}{(2^w + e^w)^2}$

26. $f'(t) = \dfrac{4-2t^2-t^3}{t^5}$

31. $r'(\theta) = e^{e^{\theta}+e^{-\theta}}(e^{\theta} - e^{-\theta})$

42. $\dfrac{dy}{d\theta} = e^{\theta-1}$

45. $\dfrac{dy}{dx} = 5^x \ln(5)$

46. $f'(x) = -\dfrac{2a^2x}{\left(a^2+x^2\right)^2}$

47. $w'(r) = \dfrac{2abr - ar^4}{\left(b+r^3\right)^2}$

48. $f'(s) = -\dfrac{3a^2s+s^3}{\left(a^2+s^2\right)^{3/2}}$

51. $g'(w) = \dfrac{20w}{\left(a^2-w^2\right)^3}$

52. $\dfrac{dy}{dx} = \dfrac{e^{2x}(2x^2-2x+2)}{\left(x^2+1\right)^2}$

53. $g'(u) = \dfrac{ae^{au}}{a^2+b^2}$

54. $\dfrac{dy}{dx} = \dfrac{2a}{\left(e^{ax}+e^{-ax}\right)^2}$

58. $g'(y) = 6y^2 e^{y^3 + 2e^{(y^3)}}$

59. $\dfrac{dy}{dx} = 18x^2 + 8x - 2$

60. $g'(z) = 5z^4 + 20z^3 - 1$

61. $f'(z) = 2\ln(3)z + \ln(4)e^z$

62. $\dfrac{dy}{dx} = 3 - 2 \cdot 4^x \ln(4)$

63. $f'(x) = 3x^2 + 3^x \ln(3)$

67. $\dfrac{dz}{dx} = 4s^3 - 1$

70. $f'(y) = 4^y \left(2\ln(4) - y^2 \ln(4) - 2y\right)$

73. $f'(x) = (-2x + 3x^2)(6 - 4x + x^7) + (4 - x^2 + 2x^3)(-4 + 7x^6)$

74. $h'(x) = 10x + 2 - \dfrac{4}{x^2} - \dfrac{8}{x^3}$

75. $f'(z) = \dfrac{5 + \sqrt{5}}{2z^{3/2}}(z - 1)$

81. $f'(t) = 6t^2 - 8t + 3$; $f''(t) = 12t - 8$

82. $r = 6/\sqrt{2}$

84. **(a)** $h'(1) = -6$
(b) $h'(0) = 0$
(c) $p'(0) = -2$

85. $r'(0) = 4$

86. **(a)** $h'(1) = 0$
(b) $h'(2) = 4$

87. $x \approx -1/2,\ 1,\ 5/2$

88. **(a)** $h'(-1) = -10$
(b) $p'(-1) = 4$

89. $y = -4x + 4$

90. $h'(1) \approx 1/2$

91. $k'(1) \approx 1$

92. $h'(2) \approx -2$

93. $k'(2) \approx 3/2$

94. **(a)** 1

(b) 2

(c) 3

(d) 3

(e) $-5/16$

(f) 13

95. **(a)** 11

(b) $-1/4$

(c) Cannot compute; $r'(1)$ is needed

(d) -3

96. **(a)** 20

(b) $11/9$

(c) -4

(d) -24

(e) $\sin(3) - 8\cos(3)$

97. **(a)** $y = 20x - 48$

(b) $y = \dfrac{11}{9}x - \dfrac{16}{9}$

(c) $y = -4x + 20$

(d) $y = -24x + 57$

(e) $y = (\sin(3) - 8\cos(3))x + 16\cos(3)$

Selected Answers for the Additional Review Problems

2. (a) (i) $50\,\dfrac{\text{miles}}{\text{hour}}$; (ii) $74\,\dfrac{\text{miles}}{\text{hour}}$; (iii) $62\,\dfrac{\text{miles}}{\text{hour}}$;

(b) $0\,\dfrac{\text{miles}}{\text{hour}}$; car returns to its starting point.

3. (a) (i) $40.1\,\dfrac{\text{ft}}{\text{sec}}$; (ii) $40.01\,\dfrac{\text{ft}}{\text{sec}}$; (iii) $40.001\,\dfrac{\text{ft}}{\text{sec}}$;

(b) $40\,\dfrac{\text{ft}}{\text{sec}}$;

(c) $\displaystyle\lim_{h\to 0}\frac{s(2+h)-s(2)}{h}=\cdots=40\,\frac{\text{ft}}{\text{sec}}$.

6. (a) -1;

(b) $f'(4), \dfrac{f(4)-f(2)}{4-2}, f'(2), \dfrac{f(2)-f(1)}{2-1}, f'(0)$.

7. (a) $f'(1.5)=-\dfrac{2.02}{1.5}$ or $f'(1.5)=-\dfrac{101}{75}$; $y=-\dfrac{2.02}{1.5}(x-1.5)+3.19$;

(b) $f(x)\approx -\dfrac{2.02}{1.5}(x-1.5)+3.19$ for $x\approx 1.5$; since $1.75\approx 1.5$, we get $f(1.75)\approx -\dfrac{2.02}{1.5}(1.75-1.5)+3.19\approx 2.8533$.

8. (a) $f'(-1)=\displaystyle\lim_{h\to 0}\frac{[3(-1+h)-(-1+h)^2]-(-4)}{h}=\cdots=5$;

(b) $f'(2)=\displaystyle\lim_{h\to 0}\frac{\dfrac{4}{2+h}-2}{h}=\cdots=-1$;

(c) $f'(3)=\displaystyle\lim_{h\to 0}\frac{\sqrt{(3+h)+1}-2}{h}=\cdots=\frac{1}{4}$;

(d) $f'(1)=\displaystyle\lim_{h\to 0}\frac{\dfrac{1}{(1+h)^2}-1}{h}=\cdots=-2$.

9. $x=-3$ because tangent line is vertical; $x=-2$ because sharp corner in the graph; $x=1$ because function is discontinuous.

11. Increasing on intervals $-2<x<0$ and $3<x<\infty$ (why?); Decreasing on intervals $-\infty<x<-2$ and $0<x<3$ (why?).

12. (a) $f'(x)=\displaystyle\lim_{h\to 0}\frac{[3(x+h)-(x+h)^2]-(3x-x^2)}{h}=\cdots=3-2x$;

(b) $f'(x)=\displaystyle\lim_{h\to 0}\frac{\dfrac{4}{x+h}-\dfrac{4}{x}}{h}=\cdots=-\frac{4}{x^2}$;

(c) $f'(3) = \lim_{h \to 0} \dfrac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h} = \cdots = \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{x+1}}$;

(d) $f'(1) = \lim_{h \to 0} \dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \cdots = -\dfrac{2}{x^3}$.

13. (a) $\dfrac{\text{dollars}}{\text{t-shirt}}$;

(b) $f(500) = 125$ means "It costs \$125 to make 500 t-shirts."; $f'(500) = 0.30$ means "When making 500 t-shirts, the cost is increasing at a rate of $0.30\,\dfrac{\text{dollars}}{\text{t-shirt}}$" or "When making 500 t-shirts, the cost increases \$0.30 for each additional t-shirt."

(c) $f(550) \approx 0.30\,\dfrac{\$}{\text{t-shirt}} \cdot 25\,\text{t-shirts} + \$125 = \$132.50$.

14. (a) $\dfrac{{}^\circ\text{C}}{\text{km}}$;

(b) $g(2) = 25$ means "It is 25°C at an altitude of 2 km."; $g'(2) = -6.5$ means "At an altitude of 2 km, the temperature is decreasing at a rate of $6.6\,\dfrac{{}^\circ\text{C}}{\text{km}}$" or "At an altitude of $2\,km$, the temperature decreases $6.5°C$ for each additional kilometer in altitude."

(c) $z = g^{-1}(T)$ gives the altitude z, in kilometers, at which the temperature is T°C; $\dfrac{\text{km}}{{}^\circ\text{C}}$

16. (a) $f(-1)$ negative because $(-1, f(-1))$ is below x-axis; $f'(-1)$ positive because tangent line has positive slope; $f''(-1)$ negative because function is concave down (and $f''(-1) \neq 0$).

17. (a) $y = f(x)$ is increasing over $0 < x < 2$ because $f'(x)$ positive on this interval;

(b) $y = f(x)$ is concave up over $0 < x < 2$ because $f''(x) = (f')'(x)$ is positive on this interval.

19. (a) $v(t) = s'(t) = 36t^2 + 1$; $a(t) = v'(t) = 72t$;

(b) At time $t = 2$ seconds, the position of object is $s(2) = 98$ feet with a velocity of $v(2) = 145\,\dfrac{\text{feet}}{\text{sec}}$ and an acceleration of $a(2) = 144\,\dfrac{\text{feet}}{\text{sec}^2}$.

20. (a) $y' = 7.8x^{6.8}$;

(b) $g(t) = t^{-5}$; $g'(t) = -5t^{-6}$;

(c) $Q = x^{3/7}$; $\dfrac{dQ}{dx} = \dfrac{3}{7}x^{-4/7}$;

(d) $f(x) = x^{-1/7}$; $f'(x) = -\dfrac{1}{7}x^{-8/7}$.

21. (a) $f'(x) = 60x^4 - 6x$;

(b) $P = 2z^{-6.5} - 12$; $P' = -13z^{-7.5}$;

(c) $y = t^{1/3} - t^{-1/3} - 3t^{-3}$; $\dfrac{dy}{dt} = \dfrac{1}{3}t^{-2/3} + \dfrac{1}{3}t^{-4/3} + 9t^{-4}$;

(d) $f(x) = 3x^{-1} - 4x^{-2}$; $f'(x) = -3x^{-2} + 8x^{-3}$;

(e) $y' = 4\ln(6) \cdot 6^x - 15x^2$;

(f) $f(t) = t^{1/2} - 10e^t$; $f'(t) = \dfrac{1}{2}t^{-1/2} - 10e^t$;

(g) $\dfrac{dy}{dx} = e^x + ex^{e-1}$;

(h) $P = 8r^{-4} - \dfrac{1}{8} \cdot 4^r + 8$; $\dfrac{dP}{dr} = -32r^{-5} - \dfrac{\ln(4)}{8} \cdot 4^r$.

22. (b) $y = -\dfrac{1}{4}(x-4) - 1$.

23. (a) Solve equation $f'(x) = -2$ for x yields $x = 0$;

(b) $y = -2x - 1$.

24. $f''(x) = 2[\ln(10)]^2 \cdot 10^x$; $f''(x) = 2[\ln(10)]^2 \cdot 10^x > 0$ implies concave up.

25. In 2010, the height of the sand dune is 625 *cm* and decreasing at a rate of 30 $\dfrac{\text{cm}}{\text{year}}$.

26. $\dfrac{dV}{dt} = 35\ln(0.85)(0.85)^t$; $V(5) = 35(0.85)^5 \approx \$15.53$ thousand;

$V''(5) = 35\ln(0.85)(0.85)^5 \approx -2.52\,\dfrac{\text{thousand dollars}}{\text{year}}$. (How do we interpret this?)

27. (a) $H'(2) = 4f'(2) - g'(2) = 23$;

(b) $G'(2) = \ln(2) \cdot 2^2 - 10g'(2) = 4\ln(2) + 30$.

28. (a) $f'(x) = x^2 \cdot \ln(3)3^x + 3^x \cdot 2x$;

(b) $y' = (3x^6 - 7x + 2) \cdot (10x^9 + 40x^4) + (x^{10} + 8x^5) \cdot (18x^5 - 7)$;

(c) $y = \dfrac{9 - x^{1/3}}{x^3}$; $y' = \dfrac{x^3 \cdot (-\frac{1}{3}x^{-2/3}) - (9 - x^{1/3}) \cdot 3x^2}{[x^3]^2}$; or $y = (9 - x^{1/3})x^{-3}$; $y' = (9 - x^{1/3}) \cdot (-3x^{-4}) + x^{-3} \cdot (-\dfrac{1}{3}x^{-2/3})$;

(d) $g'(t) = \dfrac{(4e^t + 1)] \cdot 2t - t^2 \cdot 4e^t}{[4e^t + 1]^2}$;

(e) $\dfrac{dP}{dr} = 8r^7 - \dfrac{(r + 8^r) \cdot \ln(8)8^r - 8^r \cdot (1 + \ln(8)8^r)}{[r + 8^r]^2}$;

(f) $y' = 2 \cdot 10(4x^2 - 3x + 4)^9 \cdot (8x - 3)$;

(g) $f(x) = (1 + 2x^8)^{1/2}$; $f'(x) = \dfrac{1}{2}(1 + 2x^8)^{-1/2} \cdot (16x^7)$;

(h) $g(t) = \dfrac{1}{4}t^2 - e^{t^{1/2}}$; $g'(t) = \dfrac{1}{2}t - e^{t^{1/2}} \cdot (\dfrac{1}{2}t^{-1/2})$;

(i) $P' = \dfrac{e^{z^2} \cdot (50z^4) - 10z^5 \cdot (e^{z^2} \cdot (2z))}{[e^{z^2}]^2}$;

(j) $\dfrac{dv}{dt} = \dfrac{(1-e^{-t}) \cdot (e^{-t} \cdot (-1)) - e^{-t} \cdot (-e^{-t} \cdot (-1))}{[1-e^{-t}]^2}$.

29. $f'(x) = 2x \cdot e^x + e^x \cdot 2$; $f'(0) = 2$; $y = 2x$.

30. $g'(x) = 3(4x-1)^2 \cdot 4$; $f'(\frac{1}{2}) = 3$; $y = 3(x - \frac{1}{2}) + 1$.

31. After simplifying, $f'(x) = \dfrac{1}{x^2}$; $f''(x) = \dfrac{2}{x^3}$; $f''(x) = \dfrac{2}{x^3} > 0$ for $0 < x < \infty$ implies concave up.

32. $f'(x) = e^{-x^2} \cdot (-2x)$; $f''(x) = e^{-x^2} \cdot (-2) + (-2x) \cdot (e^{-x^2} \cdot (-2x))$

33. $\dfrac{dP}{dt} = 6e^{0.013t} \cdot (0.013)$; $P(10) = 6e^{0.13} \approx 6.83$ billion people, $P'(10) = 0.078e^{0.13} \approx 0.089 \dfrac{\text{billion people}}{\text{year}}$ (How do you interpret this?)

34. (a) $H'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$; $H'(1) = f(1) \cdot g'(1) + g(1) \cdot f'(1) = -37$;

(b) $P'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; $P'(1) = \dfrac{g(1) \cdot f'(1) - f(1)g'(1)}{[g(1)]^2} = \dfrac{23}{49}$

(c) $h'(x) = 3x^2 \cdot f'(x) + f(x) \cdot (6x)$; $h'(6) = 3 \cdot 6^2 \cdot f'(6) + f(6) \cdot (6 \cdot 6) = 1044$;

(d) $k'(x) = \dfrac{g(x) \cdot (8x) - 4x^2 \cdot g'(x)}{[g(x)]^2}$; $k'(6) = \dfrac{g(6) \cdot (8 \cdot 6) - 4 \cdot 6^2 \cdot g'(6)}{[g(6)]^2} = 48$;

(e) $G'(x) = (1+g(x)) \cdot (-g'(x)) + (1-g(x)) \cdot g'(x)$; $G'(6) = (1+g(6)) \cdot (-g'(6)) + (1-g(6)) \cdot g'(6) = 12$;

(f) $q'(x) = g'(f(x)) \cdot f'(x)$; $q'(1) = g'(f(1)) \cdot f'(1) = g'(6) \cdot f'(1) = 2$;

(g) $h'(x) = \dfrac{1}{2}[f(x)]^{-1/2} \cdot f'(x)$; $h'(1) = \dfrac{1}{2}[f(1)]^{-1/2} \cdot f'(1) = -\dfrac{6^{-1/2}}{2}$;

(h) $H'(x) = f'(x^{1/2}) \cdot (\frac{1}{2}x^{-1/2})$; $H'(1) = f'(1^{1/2}) \cdot (\frac{1}{2}1^{-1/2}) = -\dfrac{1}{2}$;

(i) $F'(x) = e^{5g(x)} \cdot (5g'(x))$; $F'(6) = e^{5g(6)} \cdot (5g'(6)) = -10e^{15}$;

(j) $Q'(x) = e^{-x} \cdot g'(x) + g(x) \cdot (e^{-x} \cdot (-1))$; $Q'(6) = e^{-6} \cdot g'(6) + g(6) \cdot (e^{-6} \cdot (-1)) = -2e^{-6} - 3e^{-6} = -5e^{-6}$.