 Name: ______________________________

Activity 8

We started this week by learning about linear approximations and the Mean Value Theorem. This activity should help give some examples of how both can be useful.

1. Let $f(x) = x^2$. Calculate the Local Linear Approximation, $L(x)$, for $f(x)$ at $x = a$ and determine whether $L(x)$ will give an overestimate or underestimate of $f(x)$ at $x = a$. Briefly explain your answer.

2. Small water bugs swim in groups as protection against attacks from fish. In one observational study, the number a of attacks per individual bug per hour depended on the group size, s, following the model $a = 120s^{-1.118}$. This model is appropriate for group sizes from 1 to 100.

(a) Find a linear approximation for a, in terms of s, for group sizes near 54 water bugs.

(b) Use your linear approximation to estimate the difference in the number of attacks between groups of 52 and 56 bugs. Compare your estimate with the difference obtained using the original power function.

(c) What does the difference from part (b) tell us about water bugs and fish?

3. The table below shows the water stored $S(t)$, in acre-feet, of Lake Sonoma (a reservoir in Northern California) from March 2014 until April 2015. Here t is the number of months since the start of March 2014.

t	0	1	2	3	4	5	6
$S(t)$	182,566	185,568	179,938	171,770	163,150	154,880	147,391
t	7	8	9	10	11	12	13
$S(t)$	141,146	136,553	191,296	189,093	218,354	216,019	212,740

(a) Use the table to create a linear approximation for the water stored around May 2014 and use it to approximate the water stored in January 2015.

(b) Find and use the best linear approximation available to estimate the water stored in January 2016.

4. Recall that the Mean Value Theorem states that if a function f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is some x-value c in the interval (a, b) satisfying $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. In this problem we will use the Mean Value Theorem to show that if $f'(x)$ is positive for all x in some interval, then f is increasing on that interval.

(a) To show that a function f is increasing on an interval, we must show that if the x-values $x_1 < x_2$ are in the interval, then $f(x_1) < f(x_2)$ (i.e. as x moves from left to right, the graph of f goes up). Now, assume that f is some function whose derivative f' is positive on the interval (a, b). As a first step, use the Mean Value Theorem to show that for any two such x-values (i.e. $a < x_1 < x_2 < b$), there is some c between x_1 and x_2 so that $f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$.

(b) Use part (a) to show that if $f'(x)$ is positive on the interval (a, b) and if $a < x_1 < x_2 < b$, we have that $f(x_2) > f(x_1)$, i.e. that f is increasing.