MATH 1450 **Name:** ____________________

Activity 7

We now have a comprehensive list of how to take the derivative of basically any function we can think of. However, certain functions are still difficult to handle, e.g. if we have to do the product rule inside the quotient rule or/and vice versa. This activity will develop a new differentiation tool based on the idea of implicit differentiation, *logarithmic differentiation*, to help with some of these extreme circumstances.

Recall:
$$\ln(AB) = \ln(A) + \ln(B)$$
$$\ln(A/B) = \ln(A) - \ln(B)$$
$$\ln(A^p) = p\ln(A)$$

1. For any given function $f(x)$, find $\frac{d}{dx}\big[\ln\big(f(x)\big)\big]$. Then solve the resulting equation for $f'(x)$.

2. Let $f(x) = x^{10}e^{4x}\sin(x)$.

(a) Write out and simplify $\ln\big(f(x)\big)$ as much as possible.

(b) Differentiate the expression from part (a). This will give you $\frac{d}{dx}\big[\ln\big(f(x)\big)\big]$.

(c) Now use the results from #1 and from part (b) to find $f'(x)$.

The process used to find $f'(x)$ in #2 is what we call *logarithmic differentiation.* Use logarithmic differentiation to differentiate the following functions.

3. $s(t) = \sqrt[7]{t^4 e^{3t}}$

4. $p(x) = \dfrac{5^{x^2} \cdot \sqrt{x}^{\,x}}{e^x + 4}$

5. $g(x) = \left(\dfrac{4x^3 e^{2x}}{x^{\sin(2x)} \cos(x)} \right)^8$