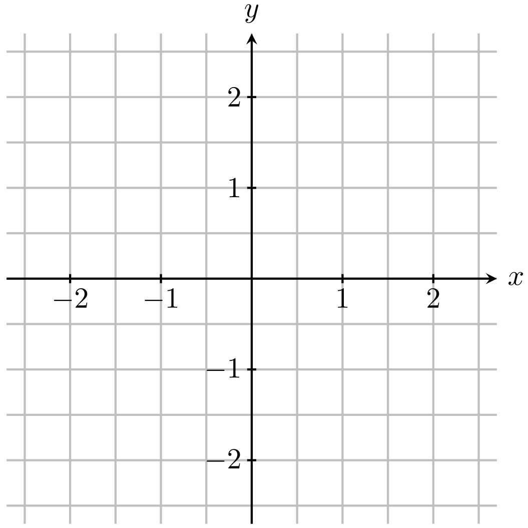
MATH 1450 **Name:** ____________________

Activity 11

In lecture, we've been discussing how to parameterize different curves. In this activity, we'll see some ways of parameterizing circles and hyperbolas and relate the two by introducing the hyperbolic trigonometric functions.

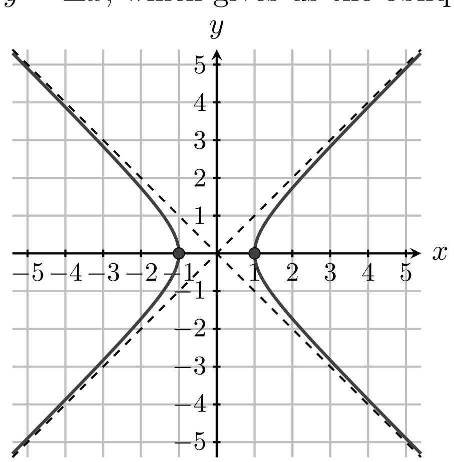
1. Graph the unit circle $x^2 + y^2 = 1$ on the axes below. Then write parameterizations of the circle which have the properties described below.



(a) The parameterization starts at $(1, 0)$ and moves counterclockwise

(b) The parameterization starts at $(0, 1)$ and moves clockwise

The graph of the hyperbola $x^2 - y^2 = 1$ is shown below. Note that as x^2 is the positive term in the expression, the hyperbola "opens" horizontally along the x-axis. Moreover, since we have $y^2 \geq 0$ for all y, we see that $x^2 \geq 1$, i.e. x cannot be between 0 and 1. Finally, as $x \to \pm\infty$, the 1 becomes nearly irrelevant and we get $x^2 \approx y^2$, i.e. $y \approx \pm x$, which gives us the oblique asymptotes.



2. Show that the curve with parametric equations $x = \dfrac{e^t + e^{-t}}{2}$, $y = \dfrac{e^t - e^{-t}}{2}$ gives a parametrization of the right half of the hyperbola $x^2 - y^2 = 1$. In which direction does the parameterization move?

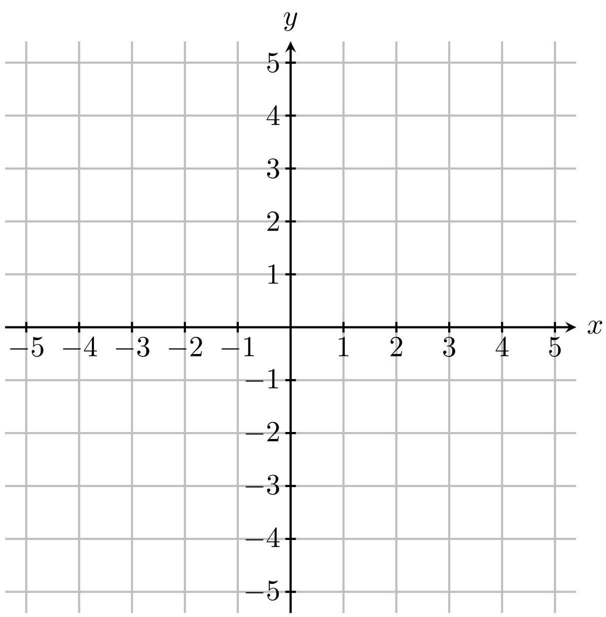
This parametrization leads to the definition of two important functions, the *hyperbolic sine* and *hyperbolic cosine* functions:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

Note that the above problem shows that $\cosh^2 x - \sinh^2 x = 1$.

3. Compute $\dfrac{d}{dx}\Big(\sinh x\Big)$ and $\dfrac{d}{dx}\Big(\cosh x\Big)$.

4. Sketch and find parametric equations for the hyperbola $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$. (Your answer should have two different sets of parametric equations!)



5. Write a sentence (or two) which explain(s) how the process of parameterizing a hyperbola of the form $\pm\dfrac{x^2}{a^2} \mp \dfrac{y^2}{b^2} = 1$ is similar to the process of parameterizing an ellipse of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. What extra step(s) are required for the hyperbola that aren't necessary for the ellipse?