MATH 2100 – HOMEWORK 5

Fall 2024

due Wednesday, **November 13**, at the beginning of class

Sections 2.5, 3.1, 3.2

This homework assignment was written in LaTeX. You can find the source code on the course website.

⋆ It is not permitted to use any AI tools or Large Language Models (ChatGPT, Claude, Gemini, etc) to assist with this assignment. ⋆

Please read the syllabus to remind yourself of our collaboration policy.

Instructions: This assignment is due at the *beginning* of class. It may be handwritten (as long as I can read it) or typed with software such as Word or Latex. Please write the questions in the correct order. Explain all reasoning.

You are no longer required to show your scratch work. However, doing scratch work on another paper before trying to write your proof is still the way to succeed on these problems!

1. Prove that any real number r that makes the equation $r - \frac{1}{r} = 5$ true must be irrational.
2. Prove that if $a + b + c \geq 35$, then either $a \geq 10$, $b \geq 12$, or $c \geq 13$.
3. Use Venn Diagrams to determine whether the equation below is true:
$$(B \cup (A \smallsetminus C)) \cap A \qquad = \qquad A \smallsetminus (A \cap \overline{B} \cap C)$$
4. Use Venn Diagrams to determine whether the equation below is true:
$$(\overline{A \cup B}) \cup (\overline{A \cup C}) \qquad = \qquad (B \cup C) \smallsetminus A$$
5. Write each of the following sets in set-builder notation.
   (a) The set S of integers that are multiples of 3 and a perfect square.
   (b) The set T of positive integers that are bigger than 10 and whose ones digit is a 5.
   (c) The set R of real numbers whose square is a rational number.
6. List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, justify how you know you've listed all of the elements).
   (a) $A = \{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$
   (b) $B = \{S \subseteq \{1, 2, 3, 4\}$: the sum of the elements of S is even$\}$
   (c) $C = \{q \in \mathbb{N} : q = 2k$ for some $k \in \mathbb{N}$ and $q = 2\ell + 1$ for some $\ell \in \mathbb{N}\}$
7. Write each of the following sets in set-builder notation.

(a) The set A of real numbers that are not rational numbers.

(b) The set B of rational numbers whose numerator is 1 and whose denominator is a prime number.

(c) The set C of pairs of real numbers (r_1, r_2) that add up to a natural number.

(d) The set D of subsets of the real numbers whose size is 10 or less.

8. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

   For all sets A, B, and C: if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

9. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

   For all sets A and B: $(A \times A) \setminus (B \times B) = (A \setminus B) \times (A \setminus B)$.

10. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

    For any two sets A and B: $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.