MATH 2100 – HOMEWORK 4

Fall 2024

due Wednesday, **October 30**, at the beginning of class

Sections 2.3, 2.4, some 2.5

This homework assignment was written in LaTeX. You can find the source code on the course website.

⋆ It is not permitted to use any AI tools or Large Language Models (ChatGPT, Claude, Gemini, etc) to assist with this assignment. ⋆

Please read the syllabus to remind yourself of our collaboration policy.

Instructions: This assignment is due at the *beginning* of class. It may be handwritten (as long as I can read it) or typed with software such as Word or Latex. Please write the questions in the correct order. Explain all reasoning.

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1. Prove that for all positive integers n,
$$\sum_{k=0}^{n}(k \cdot k!) = (n+1)! - 1.$$
2. Prove that $\displaystyle\sum_{i=1}^{n} \frac{1}{(i)(i+1)} = \frac{n}{n+1}$ for all $n \geq 1$.
3. Prove that for all $n \in \mathbb{N}$, the number $9^n - 1$ is divisible by 8.
4. Prove that for all positive integers $n \geq 4$,
$$n! > 2^n.$$
5. Use induction to prove that for all integers $n \geq 0$, the quantity $2^{2n+1} + 5^{2n+1}$ is divisible by 7.
6. Prove that $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.
7. Prove that at a completely full Milwaukee Bucks game at the Fiserv Forum, there *must* be at least two people that have both the same birthday *and* the same first initial. (Note: you will have to look up the capacity of the arena!)
8. Use the pigeonhole principle to prove that given any five integers, there will be two that have a sum or difference divisible by 7.
9. Prove that if any five points other than $(0,0)$ are placed on the coordinate plane, then there are two points, call them A and B, such that the angle formed by the rays from $(0,0)$ to A and from $(0,0)$ to B is acute.