MATH 2100 – HOMEWORK 5

Fall 2022

due Wednesday, **November 16**, at the start of class

Sections 3.1, 3.2, 3.3

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Please write the questions in the correct order. Explain all reasoning.

1. Use Venn Diagrams to determine whether the equation below is true:

$$(B \cup (A \setminus C)) \cap A \qquad = \qquad A \setminus (A \cap \overline{B} \cap C)$$

2. Use Venn Diagrams to determine whether the equation below is true:

$$(\overline{A \cup B}) \cup (\overline{A \cup C}) \qquad = \qquad (B \cup C) \setminus A$$

3. Write each of the following sets in set-builder notation.
   (a) The set S of integers that are multiples of 3 and a perfect square.
   (b) The set T of positive integers that are bigger than 10 and whose ones digit is a 5.
   (c) The set R of real numbers whose square is a rational number.

4. List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, justify how you know you've listed all of the elements).
   (a) $A = \{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$
   (b) $B = \{S \subseteq \{1,2,3,4\}$: the sum of the elements of S is even$\}$
   (c) $C = \{q \in \mathbb{N} : q = 2k$ for some $k \in \mathbb{N}$ and $q = 2\ell + 1$ for some $\ell \in \mathbb{N}\}$

5. Write each of the following sets in set-builder notation.
   (a) The set A of real numbers that are not rational numbers.
   (b) The set B of rational numbers whose numerator is 1 and whose denominator is a prime number.
   (c) The set C of pairs of real numbers (r_1, r_2) that add up to a natural number.
   (d) The set D of subsets of the real numbers whose size is 10 or less.

6. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A, B, and C: if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

7. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

$$\text{For all sets } A \text{ and } B\text{: } (A \times A) \setminus (B \times B) = (A \setminus B) \times (A \setminus B).$$

8. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

$$\text{For any two sets } A \text{ and } B\text{: } \mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B).$$

9. Prove the following set inequality:

$$(\{n^2 - 1 : n \in \mathbb{Z}\} \cap \{2k : k \in \mathbb{N}\}) \subseteq \{4m : m \in \mathbb{Z}\}.$$

10. Prove the following set inequality:

$$(\{6k + 1 : k \in \mathbb{Z}\} \cup \{6m - 1 : m \in \mathbb{Z}\}) \subseteq \{2n + 1 : n \in \mathbb{Z}\}.$$