Friday, Nov. 4 - Fall '22
Lecture #28

Skipping 3.7 + 3.8

Announcements / Reminders

* Wiley Plus #10 due next Wed (3.4, 3.5, 3.6, 3.9)
* Quiz 9 next Thursday (same)

* If you are not sure if you should stay in class or withdraw, please email me and we'll talk!

Section 3.6 – The Chain Rule and Inverse Functions (mini version)

Assume you know $\frac{d}{dx}(x^2) = 2x$.

You can this to find the derivative of the inverse function of x^2.

$f(x) = \sqrt{x} = x^{1/2}$

We want to find $f'(x)$.

square both sides

$$f(x)^2 = \underbrace{(x^{1/2})^2}_{x}$$

$f(x)^2 = x$

Take the derivative of both sides.

$$\frac{d}{dx}\left((f(x))^2\right) = \frac{d}{dx}(x)$$

By the chain rule:

$$\frac{d}{dx}\left((f(x))^2\right) = 2 \cdot f(x) \cdot f'(x)$$

So, $2 \cdot f(x) \cdot f'(x) = 1$

Solve for $f'(x)$.

$$f'(x) = \frac{1}{2 \cdot f(x)} = \frac{1}{2 \cdot \sqrt{x}}$$

Summary: Using the chain rule if we know the derivative of a function $h(x)$, then we can find the derivative of $h^{-1}(x)$.

Goal: Find the derivative of $\ln(x)$ using this trick.

The inverse of $\ln(x)$ is e^x. ③

Define $f(x) = \ln(x)$.

Do e to the power of each side.

$$e^{f(x)} = \underbrace{\cancel{e^{\ln(x)}}}_{x}$$

Take the deriv. of both sides:

$$\frac{d}{dx}\left(e^{f(x)}\right) = \frac{d}{dx}(x)$$

$$e^{f(x)} \cdot f'(x) = 1$$

Solve for $f'(x)$:

$$f'(x) = \frac{1}{e^{f(x)}} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$$

Fact: $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$.

Ex: Find the derivative of:

$$\ln(3x+2).$$

chain rule!

outside $= \ln(x)$
inside $= 3x+2$

$$\frac{1}{3x+2} \cdot \frac{d}{dx}(3x+2) = \frac{3}{3x+2}$$

Ex: Find the derivative of:

$$\cos(\ln(x^2))$$

$$\frac{d}{dx}\Big(\underbrace{\cos(\ln(x^2))}_{f(g(x))}\Big) = \underbrace{-\sin(\ln(x^2))}_{f'(g(x))} \cdot \underbrace{\frac{d}{dx}(\ln(x^2))}_{g'(x)}$$

$$= -\sin(\ln(x^2)) \cdot \frac{1}{x^2} \cdot \frac{d}{dx}(x^2)$$

$$= -\sin(\ln(x^2)) \cdot \frac{1}{x^{\cancel{2}}} \cdot (2\cancel{x})$$

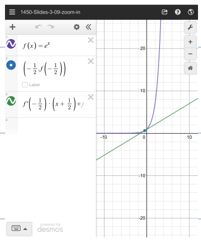
$$= \frac{-2\sin(\ln(x^2))}{x}$$

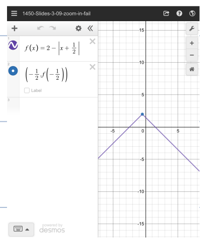
Skipping 3.7 and 3.8

Section 3.9 - Linear Approximations and The Derivative.

If you zoom in on any function at any point where it's differentiable, it looks like a straight line.

the deriv. exists





The line it looks like is the tangent line at that point.

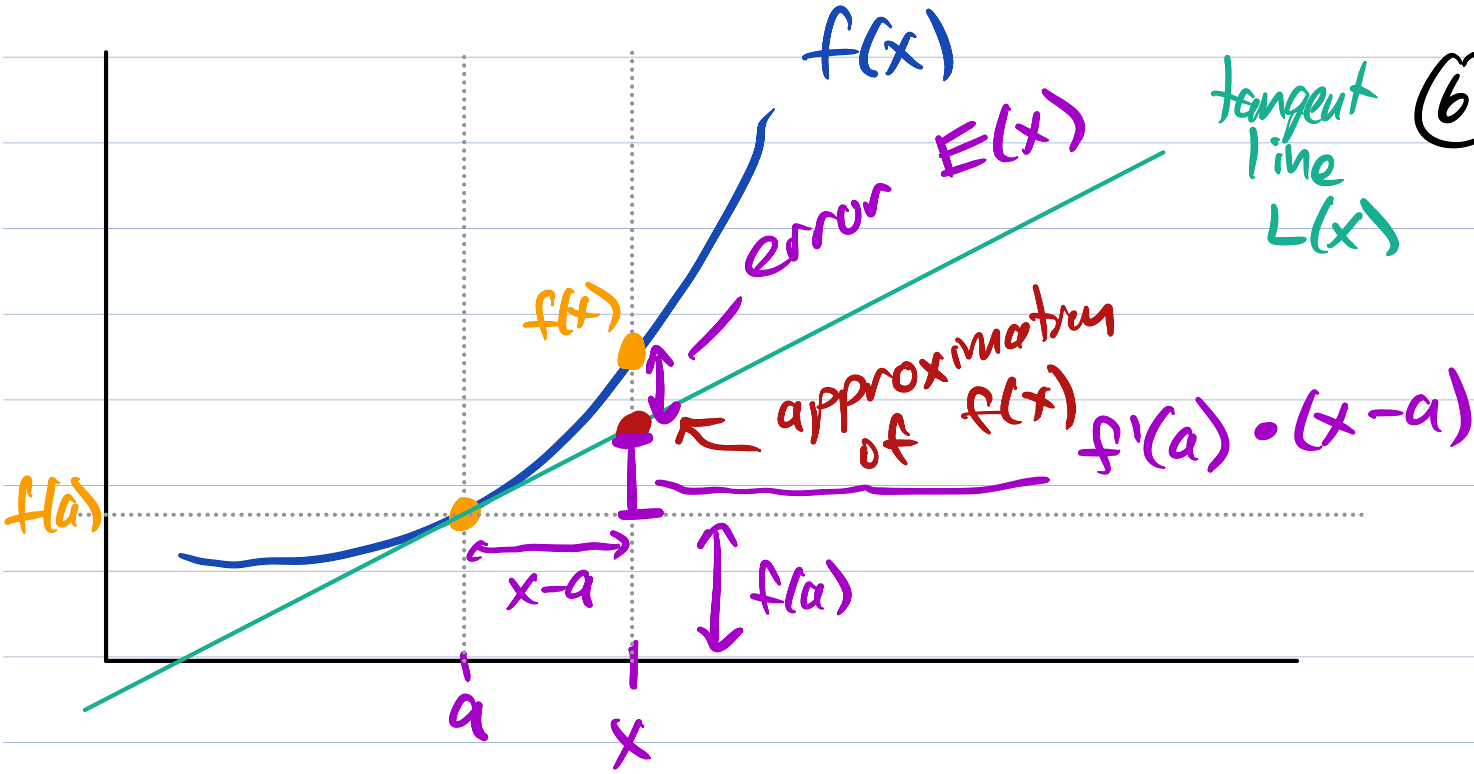
So, we can approximate the value of a function $f(x)$ near a point $x=a$ By starting at $f(a)$ and extending the tangent line.

$f(5)=2$

$f'(5)=1$

approx. $f(5.1)$

≈ 2.1

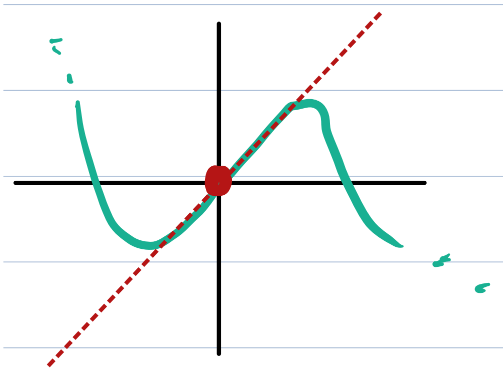


Formula for the TL:

$$f(x) \approx f(a) + (x-a) \cdot f'(a)$$

The further x gets away from a, the worse the approximation gets.
(the larger E(x) gets)

<u>Ex:</u> What is the linear approximation of $q(t) = \sin(t)$ near $t = 0$?



$$q(t) \approx q(0) + (t-0) \cdot q'(0)$$

$$q(0) = \sin(0) = 0$$

$$q'(t) = \cos(t)$$

$$q'(0) = \cos(0) = 1$$

$$\sin(t) \approx 0 + (t-0)\cdot 1$$

$$\sin(t) \approx t \quad (\text{near } t=0)$$