# MATH 2100 / 2350 – HOMEWORK 5

Fall 2020

due Wednesday, **November 11**, on D2L, by the beginning of class

Sections 2.3, 2.4, 2.5

*This homework assignment was written in LaTeX. You can find the source code on the course website.*

**Instructions:** This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit the .pdf file, but you do not have to submit the .tex file unless I ask for it** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

**Note:** *You no longer need to include your scratch work with each proof.*

1. Prove that for all positive integers $n \geq 2$, the number $2^{3n} - 1$ is not prime. (*Hint:* Proving something stronger makes this exercise easier. Prove that 7 divides $2^{3n} - 1$ for all $n \geq 2$.)
2. Prove that for all positive integers $n \geq 4$,
$$n! > 2^n.$$
3. Suppose you have an unlimited supply of 5-cent and 8-cent stamps at your disposal. Find the smallest value of $N$ so that the statement "You can use just combinations of these stamps to make $n$ cents in postage for all $n \geq N$" is true, and prove the statement by induction. (It may help to think about having multiple base cases that cover not just the first case, but the first few.)
4. Prove that for all positive integers $n$,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$
5. Prove that at a completely full Milwaukee Bucks game at the Fiserv Forum, there *must* be at least two people that have both the same birthday *and* the same first initial. (Note: you will have to look up the capacity of the arena!)
6. Use the pigeonhole principle to prove that given any five integers, there will be two that have a sum or difference divisible by 7.
7. Prove that if any five points other than $(0, 0)$ are placed on the coordinate plane, then there are two points, call them $A$ and $B$, such that the angle formed by the rays from $(0, 0)$ to $A$ and from $(0, 0)$ to $B$ is acute.
8. Use a proof by contradiction to prove that an even perfect square cannot have the form $4k + 2$.
9. Prove that if $a + b + c \geq 35$, then either $a \geq 10$, $b \geq 12$, or $c \geq 13$.