

# MATH 2100 / 2105 / 2350 – HOMEWORK 4

due Thursday, September 27, at the beginning of class

*This homework assignment was written in L<sup>A</sup>T<sub>E</sub>X. You can find the source code on the course website.*

**Instructions:** This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

**Mathematical Writing:** An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be to convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write " $3 \in S \implies 3 \notin \bar{S}$ "; instead, write "Since  $3 \in S$ , it follows that  $3 \notin \bar{S}$ ".

1. Use a truth table to determine whether  $\neg p \vee (q \wedge \neg q) \equiv \neg p$ .
2. You come across three inhabitants of an island,  $A$ ,  $B$ , and  $C$ .  $A$  says "I am lying and  $C$  is lying,"  $B$  says "Exactly one of us is lying", and  $C$  says " $B$  is telling the truth." What are the possible combinations of whether each person is lying or telling the truth? (There could be none, one, or more than one.)
3. You come across three inhabitants of an island,  $Q$ ,  $R$ , and  $S$ .  $Q$  says "The number of us that are lying is odd",  $R$  says "The number of us that are lying is even", and  $S$  says "The number of us that are lying is positive." What are the possible combinations of whether each person is lying or telling the truth? (There could be none, one, or more than one.)
4. This is a different type of island-logic-question. You might need another method to answer it. Every inhabitant of an island either always tells the truth or always lies. Each of them knows what type of inhabitant they all are, but you as an outsider do not. You come to a fork in the road and you see two inhabitants there, Alice and Bob. You want to figure out how to get to the airport. The following exchange occurs.

**You:** I would like to go the airport.

**Alice:** The airport is in the mountains or the road to the right goes to the airport.

**Bob:** The airport is in the mountains and the road to the right goes to the airport.

**Alice:** Bob is liar.

**Bob:** The road to the right goes to the airport or the airport is not in the mountains.

Which way is the airport?