

MATH 2100 / 2105 / 2350 – EXAM 1

September 27, 2018

Name: _____

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

2. You are the coach of a dodgeball club with 32 members. According to the National Dodgeball League rulebook, a full dodgeball team consists of 8 players. You would like to split your club into four teams with 8 people each, then pair up the four teams into two pairs to play against each other. In how many ways can this be done?

3. Determine whether the following statement is true or false. If true, justify why. If false, give explicit examples of sets that demonstrate it's false.

$$\text{For all sets } A \text{ and } B: |A \setminus B| = |A| - |B|.$$

4. (a) State the meaning of the notation $C(n, k)$. (It's not sufficient to just write the name. What does it signify / represent? What is its purpose?)

(b) Calculate $C(10, 4)$. Simplify your answer so it's just a positive integer.

5. Draw two Venn diagrams, one for each side of the equation below. Use your two drawings to determine whether the equation is true or false.

$$(A \setminus B) \cup (A \setminus C) = A \setminus (B \cap C)$$

6. Create a truth table for the two sides of the equation below, and use it to determine whether the equation is true or false. **State clearly how you are using your truth table to come to your conclusion.**

$$\neg t \vee (s \wedge r) \vee r \equiv \neg(\neg r \wedge t)$$

7. This time you're the coach of a dodgeball club with 18 people: 10 women and 8 men. The official National Dodgeball League rulebook says that a coed team consists of 8 people, including at least one woman and at least one man¹. How many ways can a team of 8 people with at least one woman and at least one man be chosen from a group of 10 women and 8 men?

Hint: How many ways can *any* team of 8 people be formed from 18? How many of those teams are "bad"? Use the answers to figure out how many are "good", i.e., satisfy the stipulated conditions.

¹Actually, it requires at least two women and at least two men, but I've simplified it for this question.

8. Recall that the Binomial Theorem says the following: For any positive integer n :

$$(x + y)^n = \sum_{k=0}^n C(n, k) x^k y^{n-k}.$$

Use the Binomial Theorem to find the coefficient of x^{15} in the polynomial $(x^3 - 1)^{10}$.

9. Seven couples go to the movies, and they find an empty row of 14 seats. How many ways are there for the seven couples to sit in the row such that everyone sits next to their partner?

10. You find three people, Alice, Bob, and Charlie, who say the following:

Alice: At least one of us is telling the truth.

Bob: Alice is telling the truth and I am lying.

Charlie: Bob is telling the truth and I am lying.

Who is telling the truth and who is lying?

11. **BONUS QUESTION.** Do not attempt this question until you've finished all the others.

How many ways can you split a class of $3n$ students into 3 groups that each have n students?