Growth Rates of Permutation Classes

Categorization up to the Uncountability Threshold

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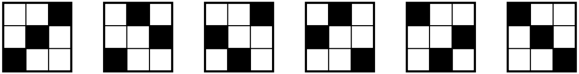

Permutation Patterns 2016

June 29, 2016

Sum-Closed Classes

Theorem. There are only countably many growth rates of permutation classes below ξ but uncountably many growth rates in every open neighborhood of it. Moreover, every growth rate of a permutation class less than ξ is achieved by a sum-closed permutation class.

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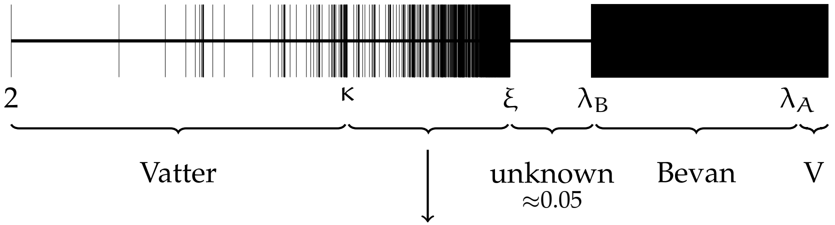
In this talk:

- identify which real numbers $< \xi$ are growth rates of permutation classes
- show that all of the growth rates are actually achieved by *finitely based* classes

Growth Rates



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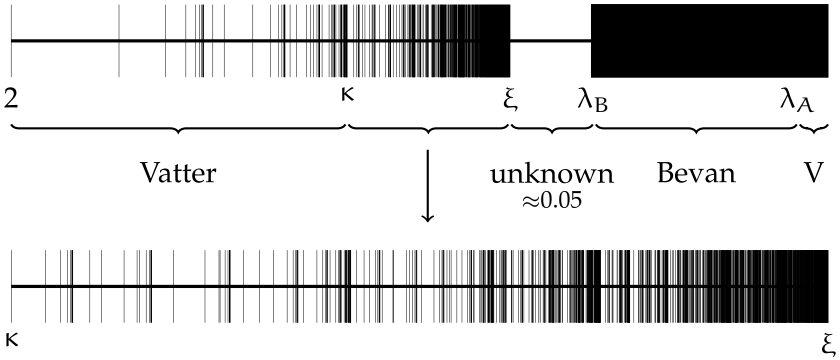

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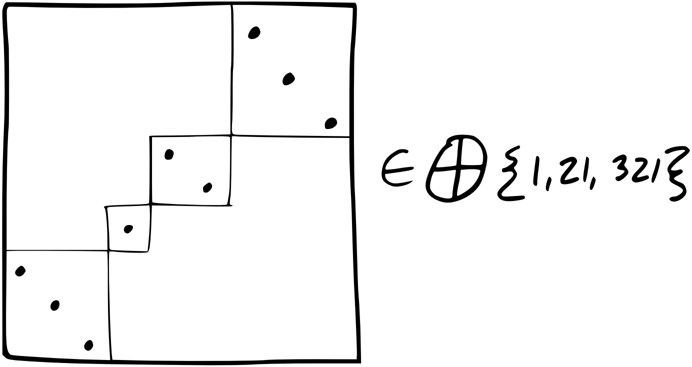

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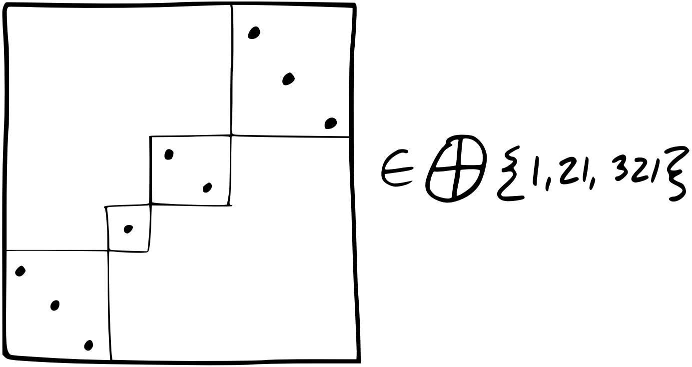

Av(231, 312, 4321) has sum-indecomposable sequence

$$1, 1, 1, 0, 0, \ldots.$$

Sum-Closed Classes

A sum-closed class $\mathcal{C}$ with sum-indecomposable sequence $\{s_i\}$ has generating function

$$\frac{1}{1 - s_1 x - s_2 x^2 - \cdots} = \frac{1}{1 - \sum_{i \geqslant 1} s_i x^i}.$$

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$$\{1, 1, 2, 2, \ldots\} \to \frac{1}{1 - x - x^2 - \sum_{i \geqslant 3} 2x^i} = \frac{1 - x}{1 - 2x - x^3}$$

$$\to \frac{1}{\text{smallest positive root of } 1 - 2x - x^3} = \kappa$$

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- Which sequences $s_1, s_2, s_3, \ldots$ are realizable by a permutation class?
- Which of those lead to a growth rate $< \xi$?

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- Other forbidden behavior. (e.g., $1, 1, 2, 3, 6$ not allowed)
- Constructions for the remaining sequences.

Tapering

Let $\mathcal{C}$ be a class with sum-indecomposable sequence $\{s_i\}$.

- For $n \geqslant 1$, if $s_n = 0$ then $s_{n+1} = 0$. (no $1, 1, \ldots, 0, 1, \ldots$)

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- For $n \geqslant 4$, if $s_n \leqslant 2$ then $s_{n+1} \leqslant 2$. (no $1, 1, \ldots, 2, 3, \ldots$)
- For $n \geqslant 5$, if $s_n \leqslant 3$ then $s_{n+1} \leqslant 3$. (no $1, 1, \ldots, 3, 4, \ldots$)

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Next one is false! Legal sum-indecomposable sequence:

$$1, 1, 2, 3, 4, 4, 5, 4, 5, 4, 5, 4, \ldots$$

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The result is that we only need to look at sequences with end behavior like

$$\ldots, 4, 4, 4, 3, 3, 2, 2, 2, 1, 1, 1, 1, 1$$

Reconstruction

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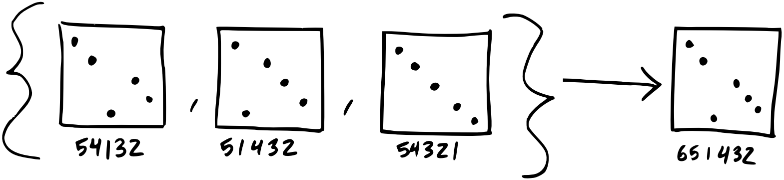

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Theorem. (P.-Vatter) Every sum-indecomposable permutation of length at least 5 that is not an increasing oscillation is uniquely determined by its set of sum-indecomposable children.

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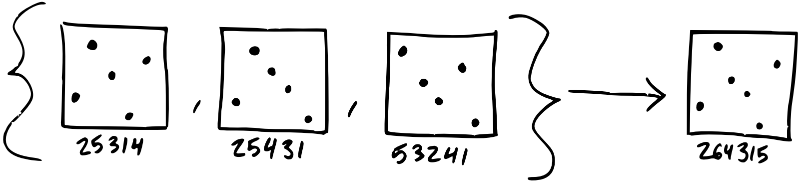

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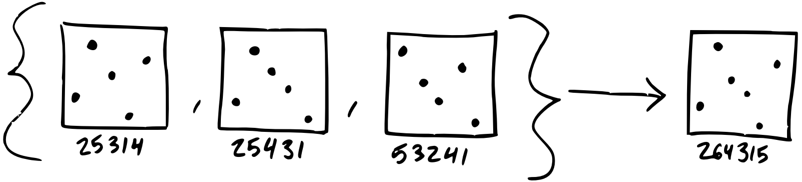

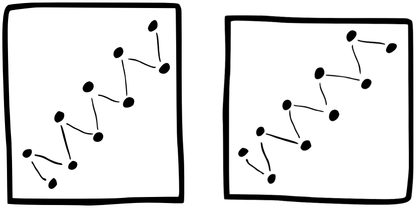

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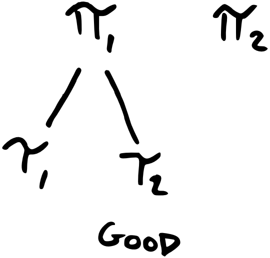

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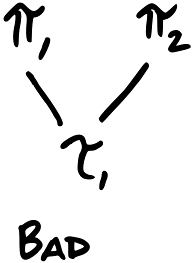

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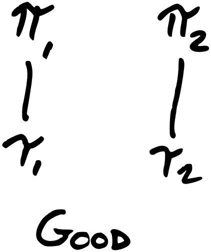

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 13$	**2.586**

Just barely too big:

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 10$	**2.509**

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 7$	**2.420**

Just barely too big:

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 4$	**2.315**

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3$	**2.275**

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$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3$	**2.275**

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3, 10$	**2.424**

Just barely too big: $\{1, 1, 3, 4\}$

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3, 5$	**2.356**

Just barely too big: $\{1, 1, 3, 4\}$

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3, 2$	**2.309**

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3, 1$	**2.292**

Just barely too big: $\{1, 1, 3, 4\}$

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞
$1, 1, 3, 3, 1$	**2.292**

Just barely too big: $\{1, 1, 3, 4\}$, $\{1, 1, 3, 3, 2\}$, $\ldots$

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Sequence	Growth Rate
$1, 1, 3, 13, 71, \ldots$	∞

Just barely too big: $\{1, 1, 3, 4\}$, $\{1, 1, 3, 3, 2\}$, $\ldots$

Growth rates below ξ: everything smaller than these.

Small Sequences

sequence	restriction	growth rate is the greatest real root of	growth rate
$1,1,3,3,1^i$	$i \leqslant 5$	$x^i(x^5-2x^4-2x^2+2)+1$	$\lesssim 2.30503$
$1,1,3,2^\infty$		x^4-2x^3-2x+1	≈ 2.29663
$1,1,3,2^i,1^\infty$		$x^i\left(x^4-2x^3-2x+1\right)+1$	$\to 2.29663$
$1,1,3,2^i,1^j$		$x^{i+j}\left(x^4-2x^3-2x+1\right)+x^j+1$	$\to 2.29663$
$1,1,2,5,2,1$		$x^6-x^5-x^4-2x^3-5x^2-2x-1$	≈ 2.30490
$1,1,2,5,2$		$x^5-x^4-x^3-2x^2-5x-2$	≈ 2.29783
$1,1,2,5,1^\infty$		$x^5-2x^4-x^2-3x+4$	≈ 2.29408
$1,1,2,5,1^i$		$x^i\left(x^5-2x^4-x^2-3x+4\right)+1$	$\to 2.29408$
$1,1,2,4,4,1^i$	$i \leqslant 5$	$x^i(x^6-2x^5-x^3-2x^2+3)+1$	$\lesssim 2.30515$
$1,1,2,4,3,3,2$		$x^7-x^6-x^5-2x^4-4x^3-3x^2-3x-2$	≈ 2.30394
$1,1,2,4,3,3,1^\infty$		$x^7-2x^6-x^4-2x^3+x^2+2$	≈ 2.30326
$1,1,2,4,3,3,1^i$		$x^i\left(x^7-2x^6-x^4-2x^3+x^2+2\right)+1$	$\to 2.30326$
$1,1,2,4,3,2^\infty$		$x^6-2x^5-x^3-2x^2+x+1$	≈ 2.30167
$1,1,2,4,3,2^i,1^\infty$		$x^i\left(x^6-2x^5-x^3-2x^2+x+1\right)+1$	$\to 2.30167$
$1,1,2,4,3,2^i,1^j$		$x^{i+j}\left(x^6-2x^5-x^3-2x^2+x+1\right)+x^j+1$	$\to 2.30167$
$1,1,2,4,2^\infty$		$x^5-2x^4-x^2-2x+2$	≈ 2.28563
$1,1,2,4,2^i,1^\infty$		$x^i\left(x^5-2x^4-x^2-2x+2\right)+1$	$\to 2.28563$
$1,1,2,4,2^i,1^j$		$x^{i+j}\left(x^5-2x^4-x^2-2x+2\right)+x^j+1$	$\to 2.28563$
$1,1,2^\infty$		x^3-2x^2-1	≈ 2.20557
$1,1,2^i,1^\infty$		$x^i\left(x^3-2x^2-1\right)+1$	$\to 2.20557$
$1,1,2^i,1^j$		$x^{i+j}\left(x^3-2x^2-1\right)+x^j+1$	$\to 2.20557$
1^∞		$x-2$	$=2$
1^i		$x^i(x-2)+1$	$\to 2$

Table 3: Legal realizable sequences dominated by a sequence in Table 1 leading to growth rates under ξ.

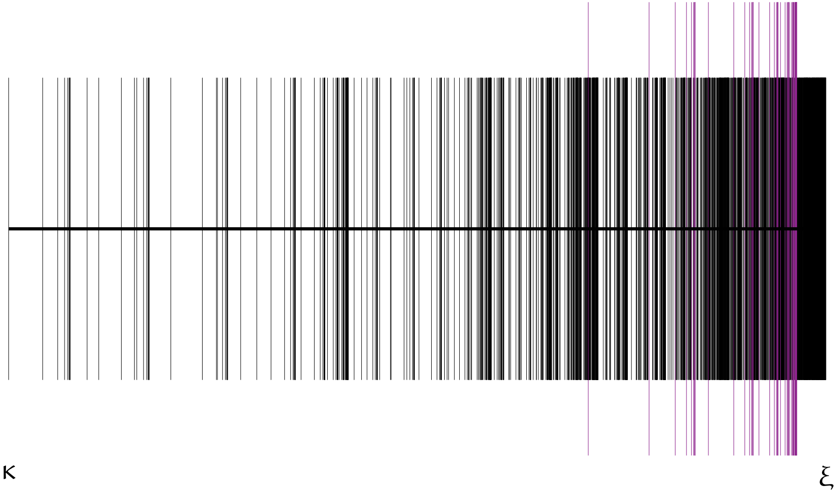
Small Sequences

sequence	restriction	growth rate is the greatest real root of
$1,1,2,3,4^i,5,4,1^j$	$i \leqslant 1,\ j \leqslant 5$	$x^{i+j+2}\left(x^5-2x^4-x^2-x-1\right)-x^j\left(x^2-x-3\right)+1$
$1,1,2,3,4^i,5,4,1^j$	i even, $j \leqslant 1$	$x^{i+j+2}\left(x^5-2x^4-x^2-x-1\right)-x^j\left(x^2-x-3\right)+1$
$1,1,2,3,4^i,5,3,3,2$	$i=1$ or i even	$x^{i+4}\left(x^5-2x^4-x^2-x-1\right)-x^4+2x^3+x+2$
$1,1,2,3,4^i,5,3,3,1^\infty$	$i \leqslant 1$	$x^{i+3}\left(x^5-2x^4-x^2-x-1\right)-x^3+2x^2+2$
$1,1,2,3,4^i,5,3,3,1^j$	$i \leqslant 1$	$x^{i+j+3}\left(x^5-2x^4-x^2-x-1\right)-x^j\left(x^3-2x^2-2\right)+1$
$1,1,2,3,4^i,5,3,3,1^j$	i even, $j \leqslant 1$	$x^{i+j+3}\left(x^5-2x^4-x^2-x-1\right)-x^j\left(x^3-2x^2-2\right)+1$
$1,1,2,3,4^i,5,3^j,2^\infty$	$i=1$ or i even, $j \leqslant 1$	$x^{i+j+1}\left(x^5-2x^4-x^2-x-1\right)-x^j(x-2)+1$
$1,1,2,3,4^i,5,3^j,2^k,1^\infty$	$i \leqslant 1,\ j \leqslant 1$	$x^{i+j+k+1}\left(x^5-2x^4-x^2-x-1\right)-x^{j+k}(x-2)+x^k+1$
$1,1,2,3,4^i,5,3^j,2^k,1^\ell$	$i \leqslant 1,\ j \leqslant 1$	$x^{i+j+k+\ell+1}\left(x^5-2x^4-x^2-x-1\right)$ $-x^{j+k+\ell}(x-2)+x^{k+\ell}+x^\ell+1$
$1,1,2,3,4^i,5,3^j,2^k,1^\ell$	i even, $j \leqslant 1,\ \ell \leqslant 1$	$x^{i+j+k+\ell+1}\left(x^5-2x^4-x^2-x-1\right)$ $-x^{j+k+\ell}(x-2)+x^{k+\ell}+x^\ell+1$
$1,1,2,3,4^i,3^\infty$		$x^i\left(x^5-2x^4-x^2-x-1\right)+1$
$1,1,2,3,4^i,3^j,2^\infty$		$x^{i+j}\left(x^5-2x^4-x^2-x-1\right)+x^j+1$
$1,1,2,3,4^i,3^j,2^k,1^\infty$		$x^{i+j+k}\left(x^5-2x^4-x^2-x-1\right)+x^{j+k}+x^k+1$
$1,1,2,3,4^i,3^j,2^k,1^\ell$		$x^{i+j+k+\ell}\left(x^5-2x^4-x^2-x-1\right)+x^{j+k+\ell}+x^{k+\ell}+x^\ell+1$

Table 4: More legal sequences. Variables that are not specified are allowed to be arbitrary nonnegative integers. The sequence of growth rates in each row of this table for which i is not bounded converge to ξ as $i \to \infty$.

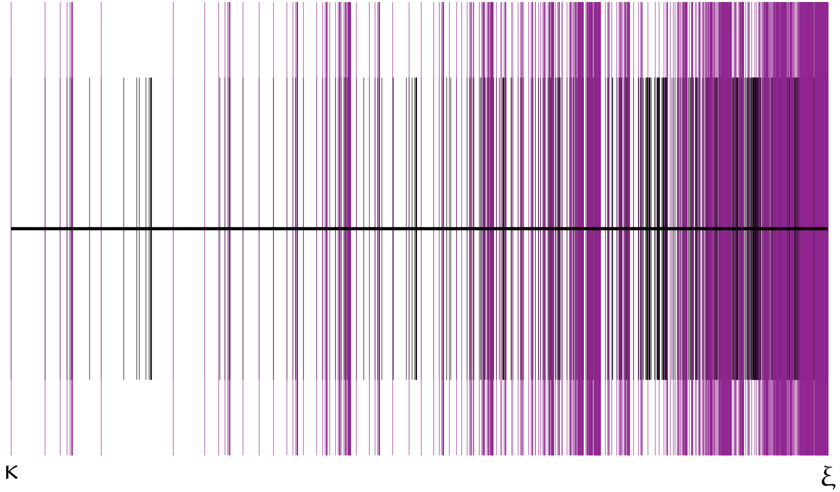
Examples

$$1, 1, 2, 4, 3, 2^i, 1^j$$



Examples

$$1, 1, 2, 3, 4^i, 3^j, 2^k, 1^\ell$$



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False, because ξ is an accumulation point from above of the classes with sum-indecomposable sequences

$$1, 1, 2, 3, 4^{2i}, (5, 4)^*$$

Future Steps

Filling in the gap between ξ and λ_A (≈ 0.05176) seems very challenging.

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Lots of other things break above ξ, too.

Thanks!