

MATH 28 – HOMEWORK 8

due Wednesday, March 1

Don't forget that you can check many of your answers using Wolfram Alpha to compute power series expansions!

1. Find a closed-form expression for the sequence $\{a_n\}_{n \geq 0}$ that has generating function

$$f(x) = \frac{x}{1 - 5x + 6x^2}.$$

(You will need to use partial fraction decomposition.)

2. Suppose that you want to form a string of digits of total length n in the following way:
 - First, a ternary string of 0s, 1s, and 2s, (a string of length 0 is permissible)
 - then, a quaternary string of 3s, 4s, 5s, and 6s, (a string of length 0 is permissible)
 - finally, a single optional 9 at the end.

For example, 0112024645669 is one such word of length 13. The generating function for the sequence $\{a_n\}_{n \geq 0}$ of the number of ways to do this is

$$f(x) = \frac{1+x}{(1-3x)(1-4x)}.$$

Use this to find a closed-form formula for a_n .

(For two bonus points: why is this the generating function for that sequence?)