MATH 28 – HOMEWORK 7

due Wednesday, February 22

Don't forget that you can check many of your answers using Wolfram Alpha to compute power series expansions!

1. Let $A(x)$ be the generating function of the sequence $\{a_n\}_{n\geq 0}$. Find the generating function for the sequence $\{b_n\}_{n\geq 0} = \{0, 0, 1, a_0, a_1, a_2, \ldots\}$. (In other words, $b_0 = 0$, $b_1 = 0$, $b_2 = 1$, and $b_n = a_{n-3}$ for $n \geq 3$.)

2. Let $A(x)$ be the generating function of the sequence $\{a_n\}_{n\geq 0}$. Prove that

$$P(x) = \frac{A(x)}{1-x}$$

   is the generating function for the sequence $\{a_0,\ a_0 + a_1,\ a_0 + a_1 + a_2, \ldots\}$ of partial sums of $\{a_n\}_{n\geq 0}$.

3. Consider the sequence $\{a_n\}_{n\geq 0}$ defined by the recurrence $a_n = \alpha a_{n-1} + \beta$, with initial condition $a_0 = 0$. Find the generating function $A(x)$ for the sequence. (Your answer will involve the variables x, α, β.)

   Use your answer to this question to find the generating function for the recurrence $a_n = 2a_{n-1} + 1$ with $a_0 = 0$, which you showed in a previous homework computes the minimum number of moves needed to solve the Towers of Hanoi problem with n disks.

4. ~~Find a closed-form expression for the sequence $\{a_n\}_{n\geq 0}$ that has generating function~~

$$f(x) = \frac{x}{1 - 5x + 6x^2}.$$

   ~~(You will need to use partial fraction decomposition.)~~