

MATH 28 – HOMEWORK 4

due Wednesday, February 1

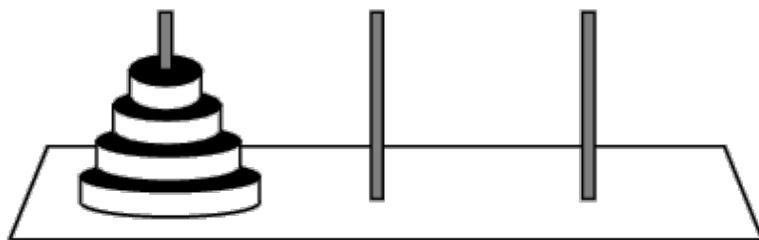
1. Submit the examples I gave you in class (also posted on the website) of five incorrect induction proofs. On a printed copy of the proofs, identify the error in each argument. You must explain what is wrong (don't just circle the error). Feel free to write your answers in the margin of the printed papers. Your answers shouldn't be too long.

Hint: If you think the base case is fine, then there must be a problem with the inductive step. Suppose the base case is $P(0)$. To help you find the problem, read the inductive step imagining $k = 0$ so that the inductive step is trying to prove $P(0) \Rightarrow P(1)$. If that doesn't seem to have a problem, then think about the $k = 1$ case, where it's proving $P(1) \Rightarrow P(2)$, etc.

2. (#77) Suppose that f is a function on the non-negative integers such that $f(0) = 0$ and $f(n) = n + f(n - 1)$. Prove that $f(n) = n(n + 1)/2$. Explain how this proves the identity

$$1 + 2 + \cdots + n = \frac{n(n + 1)}{2}.$$

3. (not in book) Use induction to prove that $2^n > n^2$ for all $n \geq 5$.
4. (#91) The "Towers of Hanoi" puzzle has three rods rising from a rectangular base with n rings of different sizes stacked in decreasing order of size on one rod. (See picture below.) A legal move consists of moving a ring from one rod to another so that it does not land on top of a smaller ring. If m_n is the smallest number of moves required to move all the rings from the initial rod to another rod that you choose, find a recurrence for m_n .



(I have a Towers of Hanoi toy in my office that you can play with during office hours to help you solve this problem.)

5. (#102, partial) The sum of the degrees of the vertices of a (finite) graph is related in a natural way to the number of edges.
 - (a) What is the relationship?
 - (b) Find a proof that this relation holds by using induction on the number of edges.
 - (c) Find a proof that this relation holds by using induction on the number of vertices.
6. (#108) Draw some trees and on the basis of your examples, make a conjecture about the relationship between the number of vertices and edges in a tree. Prove your conjecture.