MATH 28 – FINAL EXAM PRACTICE

Disclaimer: These practice problems are not to be considered comprehensive in any way. They are just some additional problems relating to the material we covered in class. You should be sure to study many other problems, including group work, homework, and the additional questions posted on the website.

Note: In every question from here on out, you need to prove, not just state, your answer.

1. Write down the generating function for the number of ways to distribute identical pieces of candy to three children so that every child gets at least 4 pieces. Write this generating function as a quotient of polynomials. Then, use the generating function to find a formula for the number of ways that n pieces of candy can be distributed. (*Hint:* Use Newton's Generalized Binomial Theorem.)

2. Consider the recurrence $a_n = a_{n-1} + 2a_{n-2}$, with initial conditions $a_0 = 1$ and $a_1 = 1$. Find the generating function $f(x) = \sum_{n \geq 0} a_n x^n$. Find a closed form formula for a_n.

3. Let
$$f(x) = \frac{2x + 5x^2}{(1-2x)(1+x^2)} = 2x + 9x^2 + 16x^3 + 27x^4 + 56x^5 + \cdots .$$
Find a closed-form formula for the coefficient of x^n in the power series expansion of $f(x)$.

4. Use Newton's Generalized Binomial Theorem to calculate the coefficient of x^n in the generating function
$$\frac{1}{\sqrt[3]{1-3x}} = (1-3x)^{-1/3}.$$

5. Use Newton's Generalized Binomial Theorem to calculate the coefficient of x^n in the generating function
$$\frac{\sqrt{1+x}}{(1-7x)^3}.$$
Your answer may contain a single summation sign.

6. Use the principle on inclusion-exclusion to prove directly that the number of set partitions of an n element set into k parts is
$$S(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n.$$
By "directly", I mean don't use the process of first counting the onto functions, like we did in group work.

7. Each person attending a party has been asked to bring a prize. The person planning the party has arranged to give out exactly as many prizes as there are guests, but any person may win any number of prizes. If there are n guests, in how many ways may the prizes be given out so that nobody gets the price that he or she brought?