MATH 28 – EXAM 2 PRACTICE

Disclaimer: These practice problems are not to be considered comprehensive in any way. They are just some additional problems relating to the material we covered in class. You should be sure to study many other problems, including group work, homework, and the additional questions posted on the website.

Note: In every question from here on out, you need to prove, not just state, your answer.

1. How many solutions are there in nonnegative integers to the equation $x_1 + x_2 + \cdots + x_m = r$, where m and r are constants?

2. Find a recurrence for the Lah numbers $L(k, n)$ similar to the one you found for the Stirling numbers.

3. How many labeled trees on n vertices have exactly 3 vertices of degree one? (The book's solution to this involves Prüfer codes. We didn't cover Prüfer codes, so find another solution.)

4. How many compositions of n are there into k parts, all of which are odd?

5. How many ways can you distribute p (identical) ping pong balls to q (distinct) children such that each child gets at least m ping pong balls?

6. How many ways can you distribute p (distinct) books to q (distinct) children such that each child gets at least 1 book?

7. Find a formula (with no summation signs) for $S(k, 2)$.

8. An integer partition is called *self-conjugate* if it is equal to its conjugate. Find a relationship between the number of self-conjugate partitions of k and the number of partitions of k into distinct odd parts.

9. Show that
$$P(k, n) \geq \frac{1}{n!}\binom{k-1}{n-1}.$$
(Hint: It's probably easiest to prove using combinatorial reasoning, rather than induction.)

10. Define $Q(k, n)$ as the number of partitions of k into n *distinct* parts (i.e., all the parts have different values). Show that
$$Q(k, n) \leq \frac{1}{n!}\binom{k-1}{n-1}.$$
(Hint: It's probably easiest to prove using combinatorial reasoning, rather than induction.)

11. Write down the generating function for the number of ways to distribute identical pieces of candy to three children so that every child gets at least 4 pieces. Write this generating function as a quotient of polynomials. ~~Then, use the generating function to find a formula for the number of ways that n pieces of candy can be distributed.~~

12. Suppose you have ten pennies, eight nickels, five dimes, and six quarters. Find the generating function $f(x)$ such that the coefficient of x^n is the number of ways you can make n cents using a subset of your coins.

13. Suppose you have an unlimited number of pennies, nickels, dimes, and quarters. Find the generating function $f(x)$ such that the coefficient of x^n is the number of ways you can make n cents using any combination of these coins.

14. Consider the recurrence $a_n = a_{n-1} + 2a_{n-2}$, with initial conditions $a_0 = 1$ and $a_1 = 1$. Find the generating function $f(x) = \sum_{n\geq 0} a_n x^n$.

15. Consider the recurrence $a_n = 5a_{n-1} - 6a_{n-2} + 2^n$ with initial conditions $a_0 = A$ and $a_1 = B$. Find the generating function $f(x) = \sum_{n\geq 0} a_n x^n$.

16. ~~Find a closed form formula for a_n as defined in Question 14 by using the generating function you found.~~

17. Suppose $f(x)$ is the generating function for the sequence $\{a_n\}$. Find the generating function of the sequence $\{na_n\}$ in terms of $f(x)$.