MATH 118, WINTER '16

HOMEWORK 2

Due Monday, Feb 1

1. Let $\mathcal{A}$ be a combinatorial class with no objects of size 0. Let $\mathcal{M} = \text{MSET}(\mathcal{A})$ and $\mathcal{P} = \text{PSET}(\mathcal{A})$. Prove **combinatorially** that $M(z) = P(z)M(z^2)$. (*Cryptic Hint:* Every positive integer is either of the form $2n$ or $2n+1$.)

2. This exercise considers two types of walks related to Dyck paths.

   a) A *meander* is a walk in the first quadrant that starts at the origin and takes steps $(1,1)$ and $(1,-1)$ such that the walk never passes below the x-axis. Unlike a Dyck path, a meander does not have to end on the x-axis. Find a symbolic construction and then an OGF for the class of meanders. (The size of a meander is the number of steps in the walk.)

   b) A *bridge* is a walk in the first and fourth quadrants that starts at the origin and takes steps $(1,1)$ and $(1,-1)$. The walk is allowed to move above and below the x-axis freely, but the walk must end on the x-axis. Find a symbolic construction and then an OGF for the class of bridges. (The size of a bridge is the number of steps in the walk.)

3. A *double surjection* of size n is a map from $[1\,..\,n]$ to $[1\,..\,r]$ for some r, such that for all $y \in [1\,..\,r]$ there exist distinct $x_1, x_2 \in [1\,..\,n]$ such that $f(x_1) = f(x_2) = y$. Find a symbolic construction and then an EGF for the class of double surjections.

4. Find a symbolic construction and then an EGF for the class of permutations with an odd number of cycles. Can you explain this result combinatorially?