MATH 31 – HOMEWORK 7

due Wednesday, August 16

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. Let X be a set (not necessarily finite) and let $Y \subseteq X$. Recall the group $(\mathcal{P}(X), \triangle)$ from Homework 1. Prove without using any material from Chapter 14 that
$$\mathcal{P}(X)/\mathcal{P}(Y) \cong \mathcal{P}(X \setminus Y).$$
(Don't forget to make sure you verify that $\mathcal{P}(Y)$ is a subgroup!)

2. (13.11) Suppose $H \lhd G$ and $K \lhd G$.
   (a) Prove that $G/H \times G/K$ has a subgroup that is isomorphic to $G/(H \cap K)$.
   (b) Prove that if $G = HK$ then $G/(H \cap K) \cong G/H \times G/K$.

3. (13.19) Let $\varphi : G \to K$ be a homomorphism. Prove that φ is injective if and only if $\ker(\varphi) = \{e_G\}$.

4. (13.22) Let $\varphi : G \to K$ be a surjective homomorphism and assume that K is abelian. Show that every subgroup of G containing $\ker(\varphi)$ is normal.

5. (14.4) Let n be a positive integer. Show that every abelian group of order n is cyclic if and only if n is not divisible by the square of any prime.