

MATH 31 – HOMEWORK 6

due Wednesday, August 9

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

- (12.13) Let $\phi : G \rightarrow H$ be a homomorphism.
 - Show that if H is abelian and ϕ is one-to-one, then G is abelian.
 - Show that if G is abelian and ϕ is onto, then H is abelian.
 - Show that if ϕ is an isomorphism then G is abelian if and only if H is.
- (12.21) Let G be the group of nonzero complex numbers under multiplication and let H be the subgroup of $GL(2, \mathbb{R})$ consisting of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where not both a and b are 0. Show that $G \cong H$.
- (13.5) Let G be the group of all real-valued functions on the real line, under addition of functions. Let H be the subset of G consisting of all f such that $f(0) = 0$.
 - Show that $H \triangleleft G$.
 - Show that $G/H \cong (\mathbb{R}, +)$.
- (13.12) Let G be a group, let $K \triangleleft G$, and let H be a subgroup of G such that $HK = G$ and $H \cap K = \{e\}$. Show that $G/K \cong H$. (Note: HK means the subgroup $\{hk : h \in H, k \in K\}$. One can prove that if H and K are subgroups of G , and if K is normal in G , then HK is a subgroup of G .)